

EE 435

Lecture 15

Compensation of Feedback Amplifiers

Thank you Nathan

Executive Summary 2/17/23

2-Stage Design Op Amps

- More possible variants
- Basic analysis from quarter circuit
- Small number of them are used

Guidelines for 2-Stage Op Amps

<u>First Stage</u>	<u>Second Stage</u>
Tail Current	Tail Voltage
Large gain	Small gain
Opposite types of transistors (p-n)	Opposite types of transistors (p-n)
Concerned with common mode input range, CMRR, noise	Concerned with output range

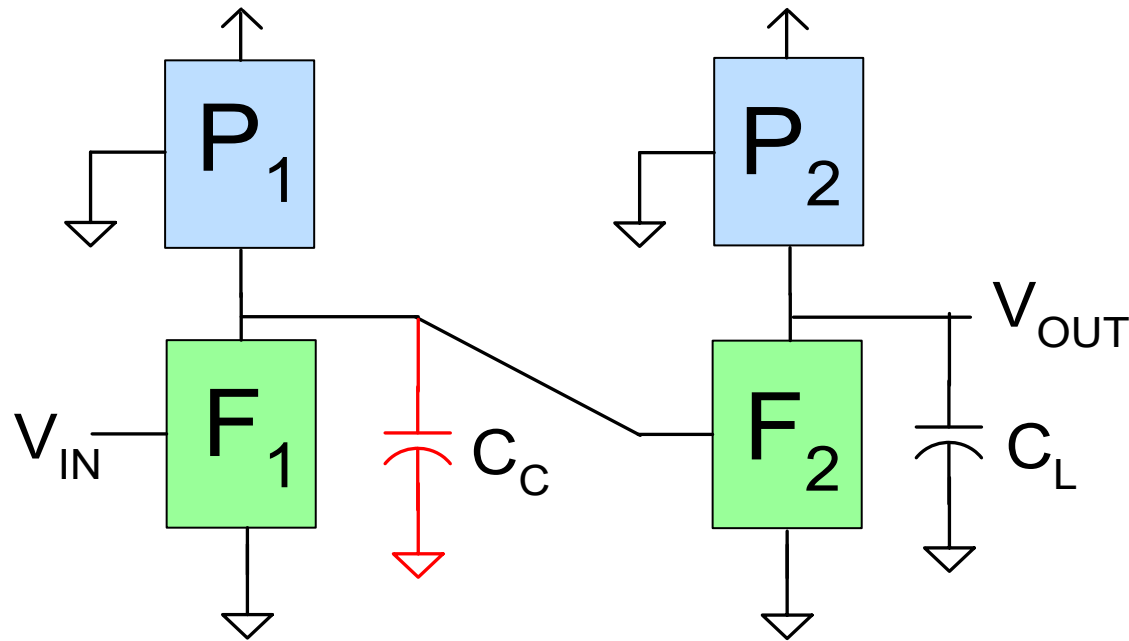
Compensation

- Compensate what is needed or face performance drop
- Should short circuit at high frequencies and pull down gain at low frequencies

Miller capacitance

- Large capacitance can be created from a small capacitor if it bridges two nodes with a large inverting gain
- Works if gain is independent of frequency
- 2nd stage amplifiers can be used to make a Miller capacitor if no circuit overhead
- Capacitance is reduced by gain of second stage
- Second stage gain isn't constant; need new analysis

Analysis of Internal Node Compensated Two-Stage Op Amps

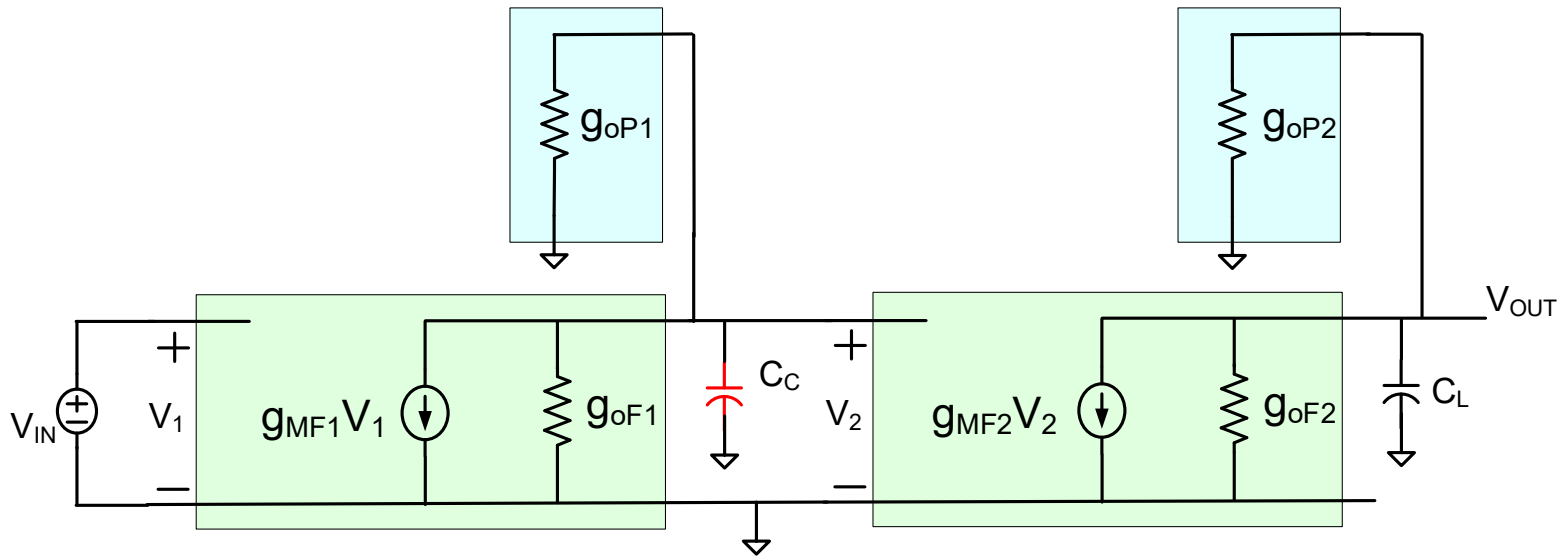


Consider single-ended input-output (differential analysis only slightly different)

Can't get everything but can get most of the small-signal results

Since internal node compensated, must have $p_1 \ll p_2$

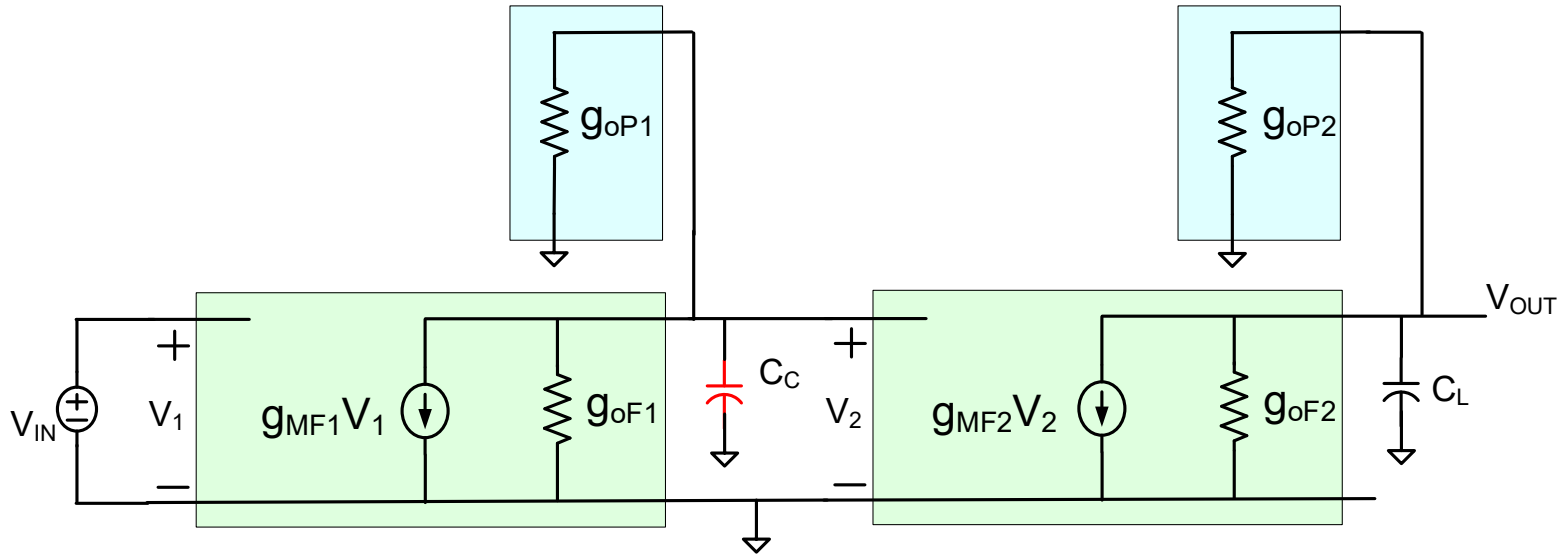
Analysis of Internal Node Compensated Two-Stage Op Amps



$$\left. \begin{aligned} V_2 (sC_C + g_{oF1} + g_{oP1}) + g_{mF1} V_{IN} &= 0 \\ V_{OUT} (sC_L + g_{oP2} + g_{oF2}) + g_{mF2} V_2 &= 0 \end{aligned} \right\}$$

$$A_V(s) = \frac{-g_{mF1}}{sC_C + g_{oF1} + g_{oP1}} \cdot \frac{-g_{mF2}}{sC_L + g_{oP2} + g_{oF2}}$$

Analysis of Internal Node Compensated Two-Stage Op Amps



$$\mathbf{A}_{V0} = \left(\frac{\mathbf{g}_{mF1}}{\mathbf{g}_{oF1} + \mathbf{g}_{oP1}} \right) \left(\frac{\mathbf{g}_{mF2}}{\mathbf{g}_{oF2} + \mathbf{g}_{oP2}} \right)$$

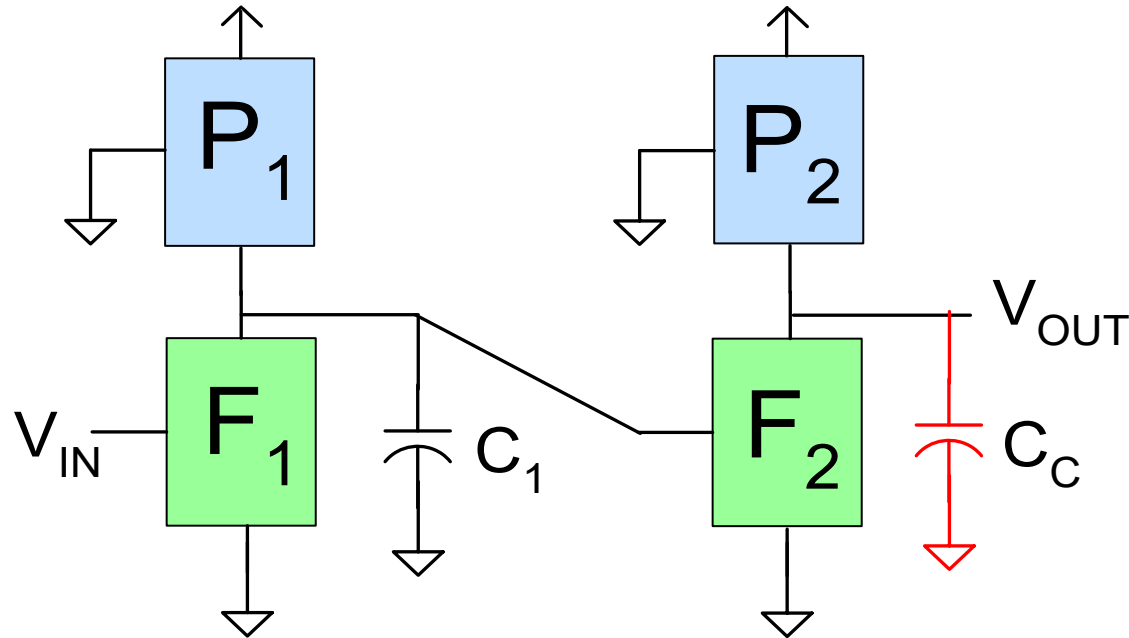
$$|\mathbf{p}_1| = \frac{(\mathbf{g}_{oF1} + \mathbf{g}_{oP1})}{\mathbf{C}_C}$$

$$|\mathbf{p}_2| = \frac{(\mathbf{g}_{oF2} + \mathbf{g}_{oP2})}{\mathbf{C}_L}$$

$$\mathbf{BW} = |\mathbf{p}_1|$$

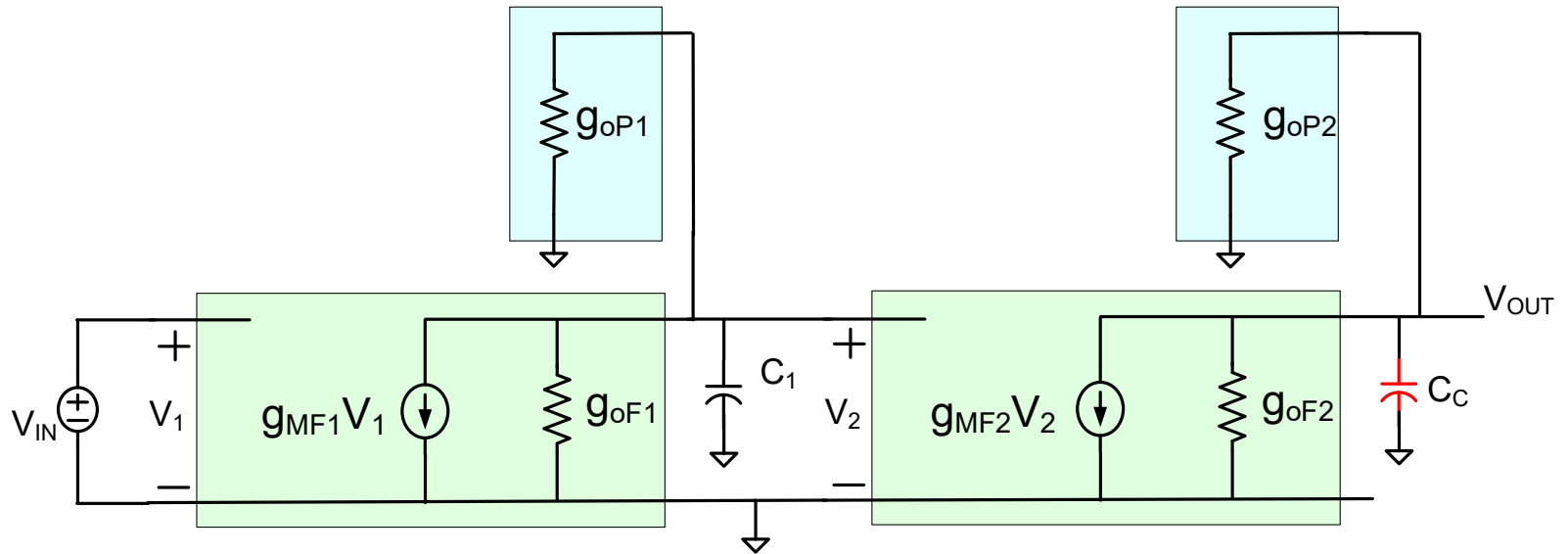
$$\mathbf{GB} = \frac{\mathbf{g}_{mF1}\mathbf{g}_{mF2}}{(\mathbf{g}_{oF2} + \mathbf{g}_{oP2})\mathbf{C}_C}$$

Analysis of Load Compensated Two-Stage Op Amps



Can't get everything but can get most of the small-signal results

Analysis of Load-Compensated Two-Stage Op Amps



$$\mathbf{A}_{V0} = \left(\frac{\mathbf{g}_{mF1}}{\mathbf{g}_{oF1} + \mathbf{g}_{oP1}} \right) \left(\frac{\mathbf{g}_{mF2}}{\mathbf{g}_{oF2} + \mathbf{g}_{oP2}} \right)$$

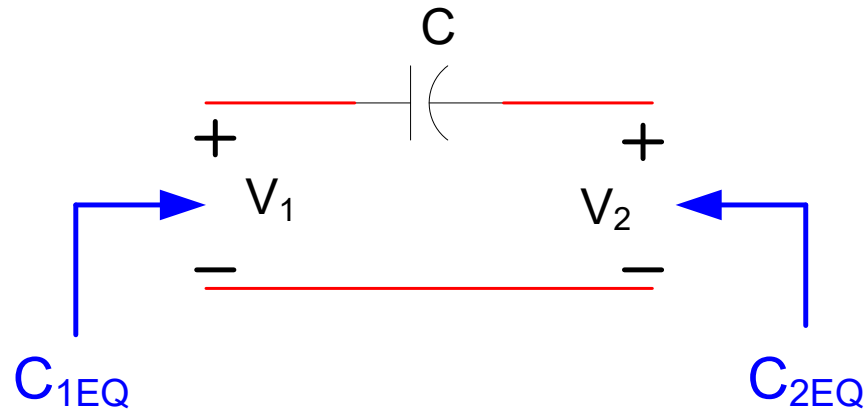
$$|\mathbf{p}_2| = \frac{(\mathbf{g}_{oF2} + \mathbf{g}_{oP2})}{\mathbf{C}_C}$$

$$|\mathbf{p}_1| = \frac{(\mathbf{g}_{oF1} + \mathbf{g}_{oP1})}{\mathbf{C}_1}$$

$$\mathbf{BW} = |\mathbf{p}_2|$$

$$\mathbf{GB} = \frac{\mathbf{g}_{mF1}\mathbf{g}_{mF2}}{(\mathbf{g}_{oF1} + \mathbf{g}_{oP1})\mathbf{C}_C}$$

Miller Capacitance - Review



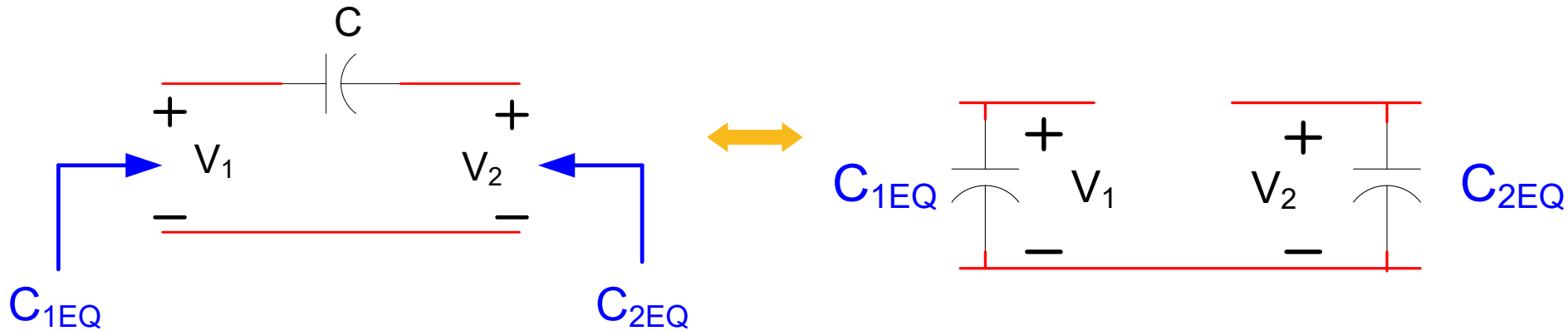
If $V_2 = -AV_1$ then for A large

$$C_{1EQ} = C(1 + A) \approx CA \quad C_{2EQ} = C\left(1 + \frac{1}{A}\right) \approx C$$

Thus, a large effective capacitance can be created with a much smaller capacitor if a capacitor bridges two nodes with a large inverting gain !!

Note: The symbol "A" used in the Miller Capacitance should not be confused with the gain of the op amp

Miller Capacitance - Review



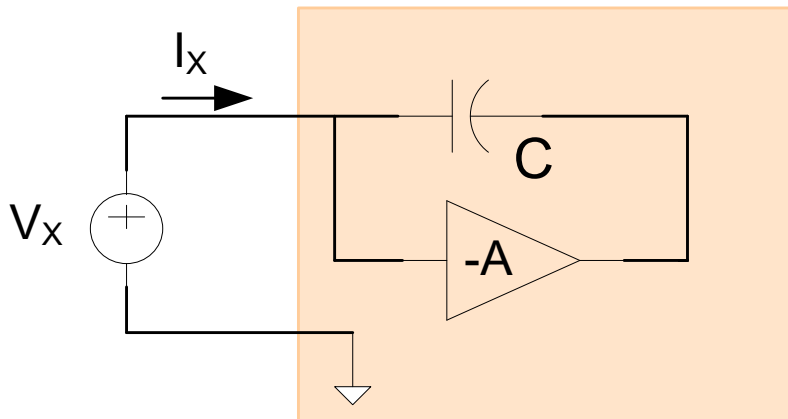
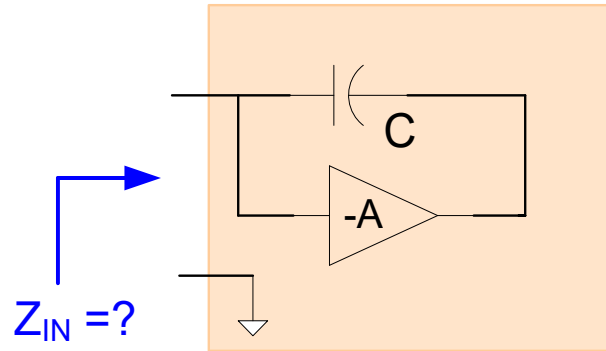
If $V_2 = -AV_1$ then for A large

$$C_{1EQ} = C(1 + A) \approx CA \qquad C_{2EQ} = C\left(1 + \frac{1}{A}\right) \approx C$$

- If A changes with frequency, C_{1EQ} and C_{2EQ} are no longer pure capacitors
- More useful for giving a concept than for accurate actual analysis because of frequency dependence of A

Miller Capacitance - Review

The Basic Concept – from capacitance multiplication



$$I_X = [V_X - (-AV_X)]sC = V_X s [C(1+A)]$$

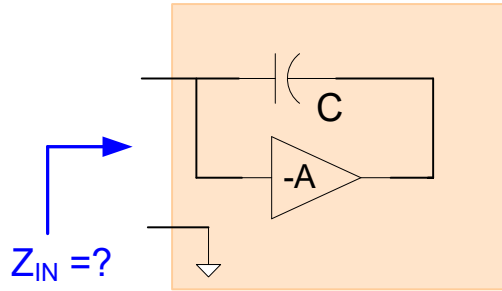
thus

$$Z_{IN} = \frac{V_X}{I_X} = \frac{1}{s [C(1+A)]}$$

So, if A is constant, input looks like a capacitor of value

$$C_{EQ} = C(1+A)$$

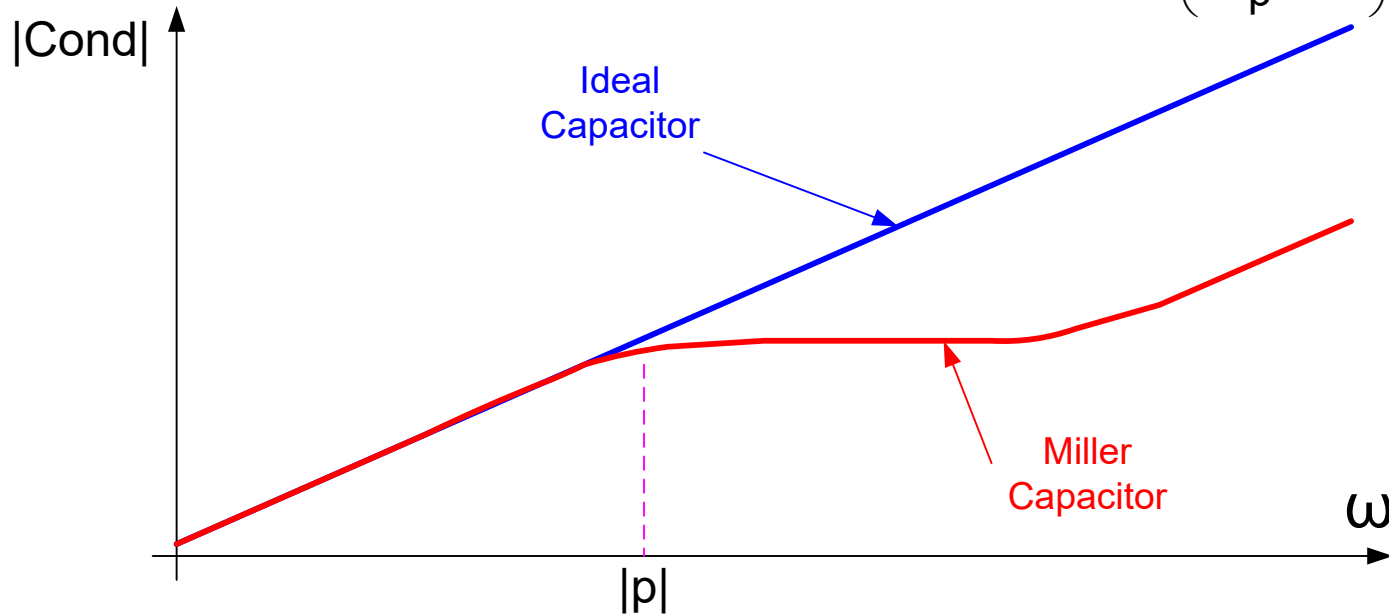
Miller Capacitance - Review



$$Z_{IN} = \frac{V_X}{I_X} = \frac{1}{s[C(1+A)]}$$

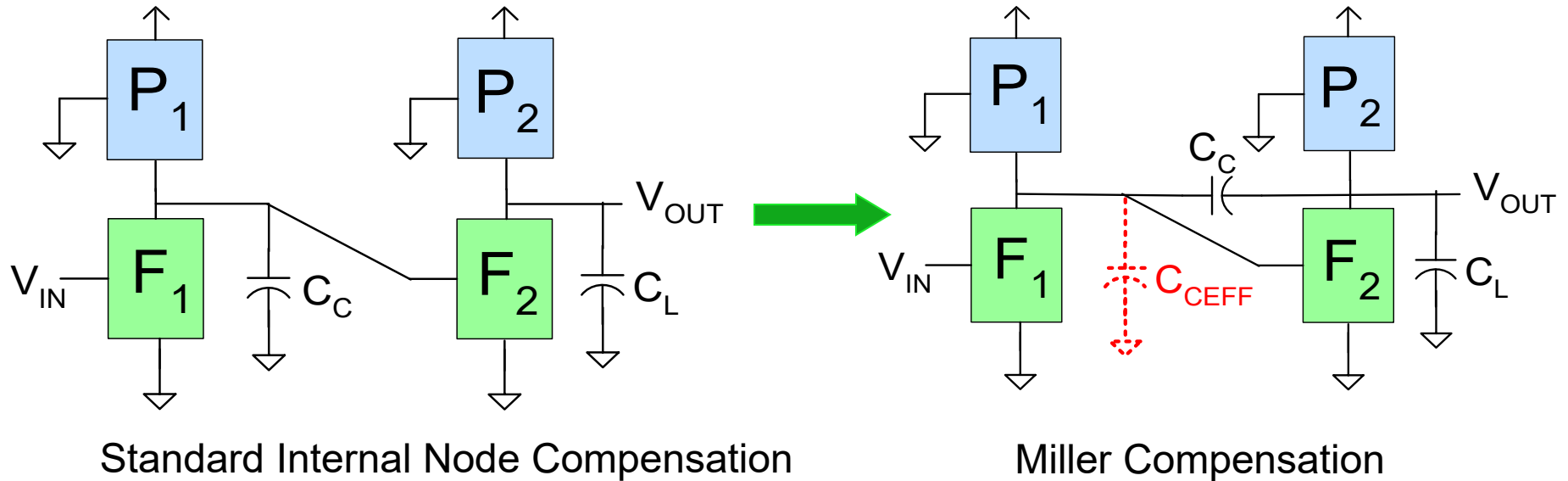
$$\text{If } A(s) = \frac{A_0}{\frac{s}{p} + 1}$$

$$G_{IN} = s[C(1+A)] = sC \left(\frac{\frac{s}{p} + 1 + A_0}{\frac{s}{p} + 1} \right)$$



Does not behave as a capacitor for $\omega > p$

Internal-Node Miller-Compensated Two-Stage Op Amp



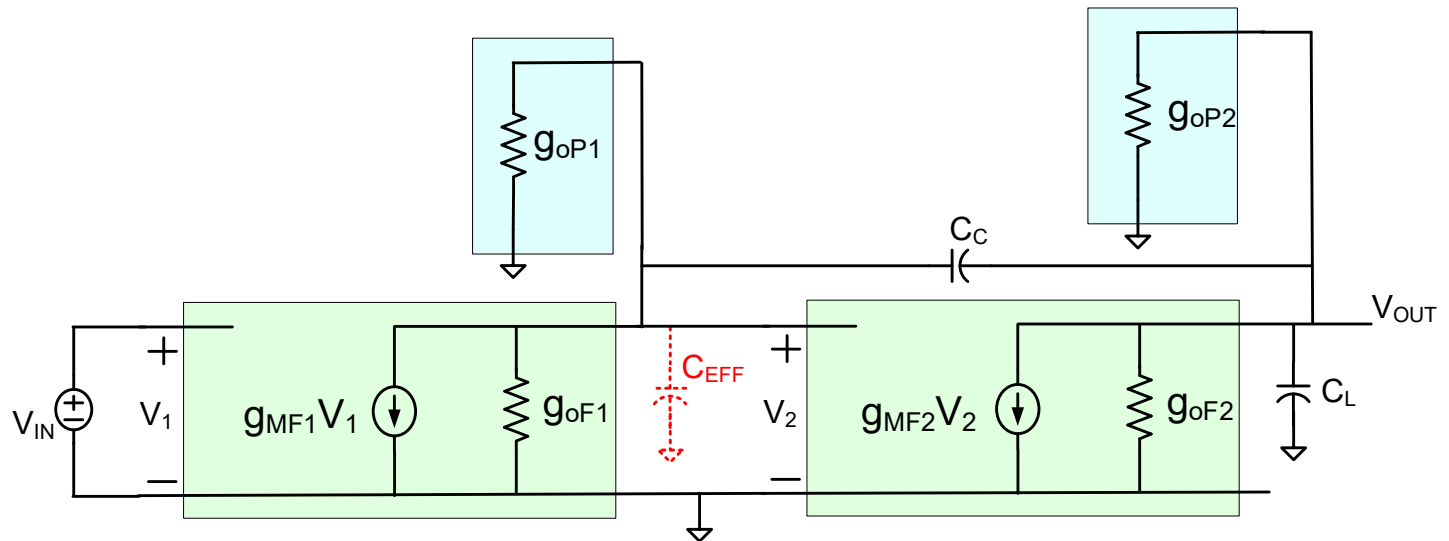
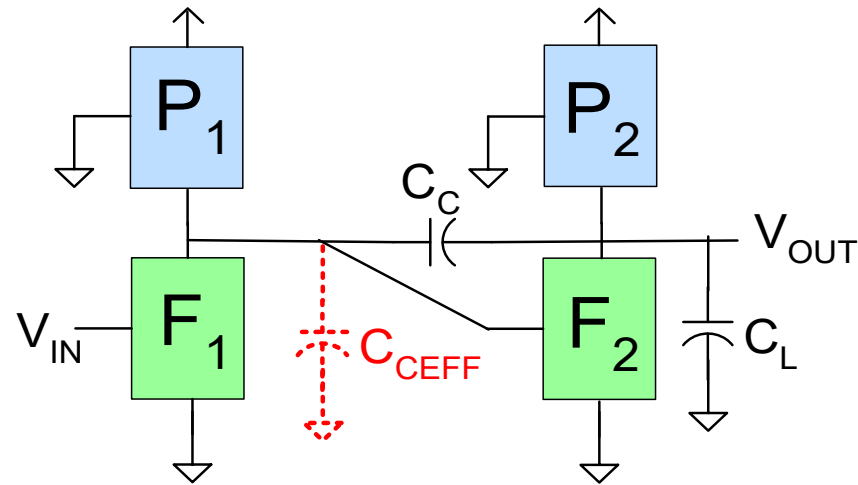
The second stage amplifier can be used to create a Miller capacitance at its input with no circuit overhead!

Compensation capacitance reduced by approximately the gain of the second stage! (the value of the two C_C 's are not the same)

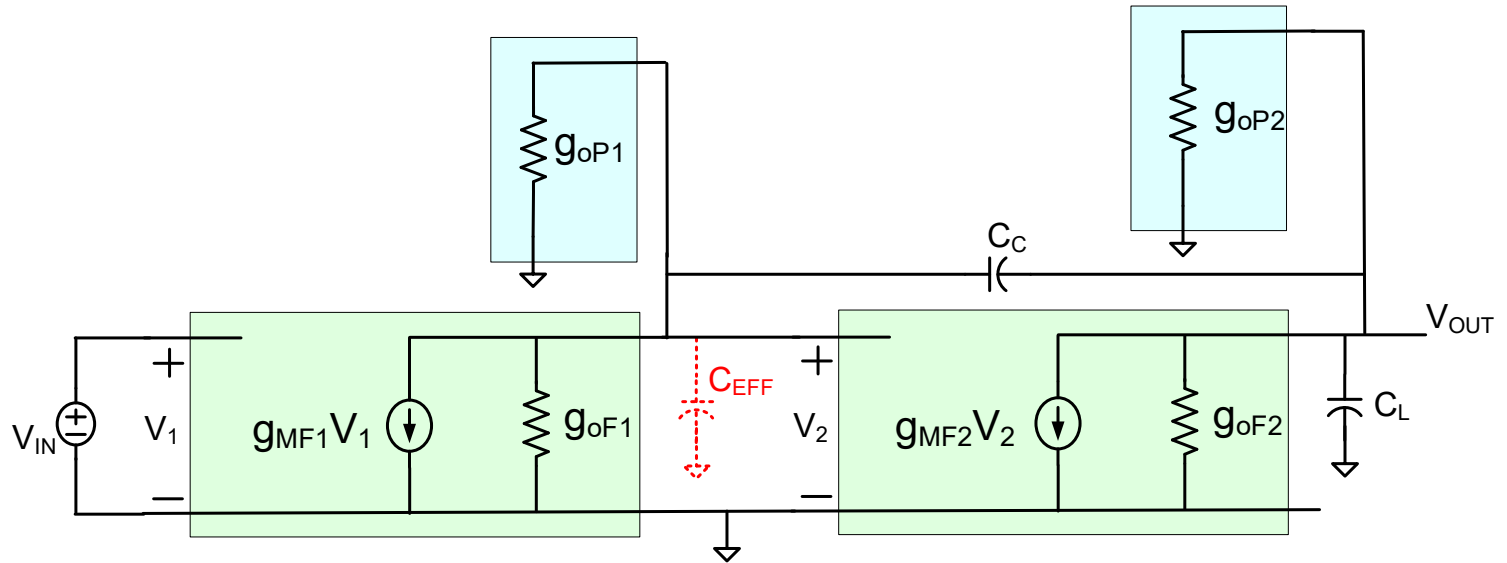
Since the gain of the second stage is not constant, however, a new analysis is needed

If C_C is small enough, this can become an internally compensated op amp with internal-node compensation

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps

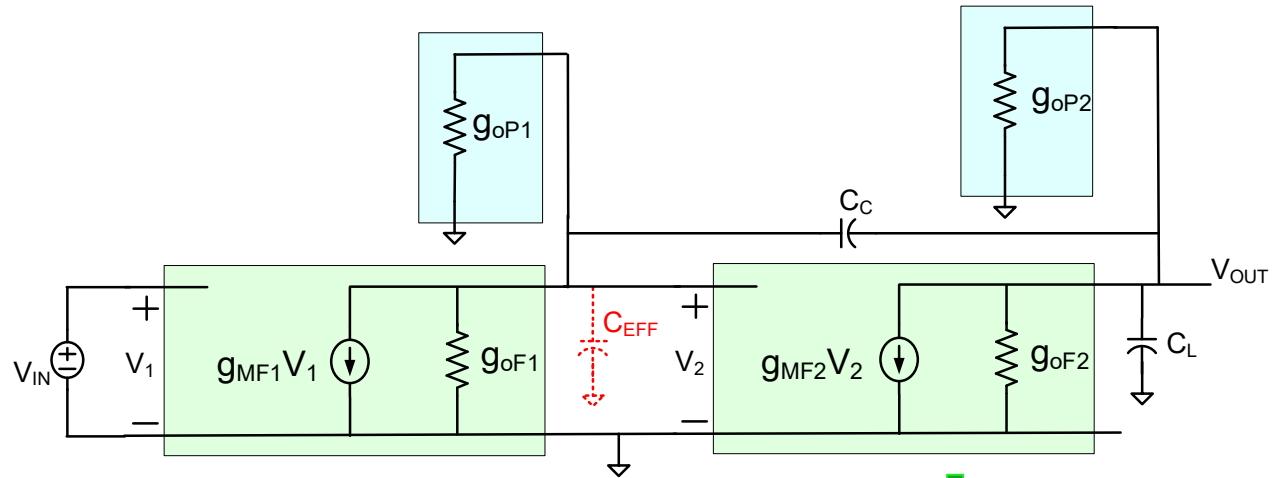


$$\mathbf{A}_{v0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}} \right) \left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}} \right)$$

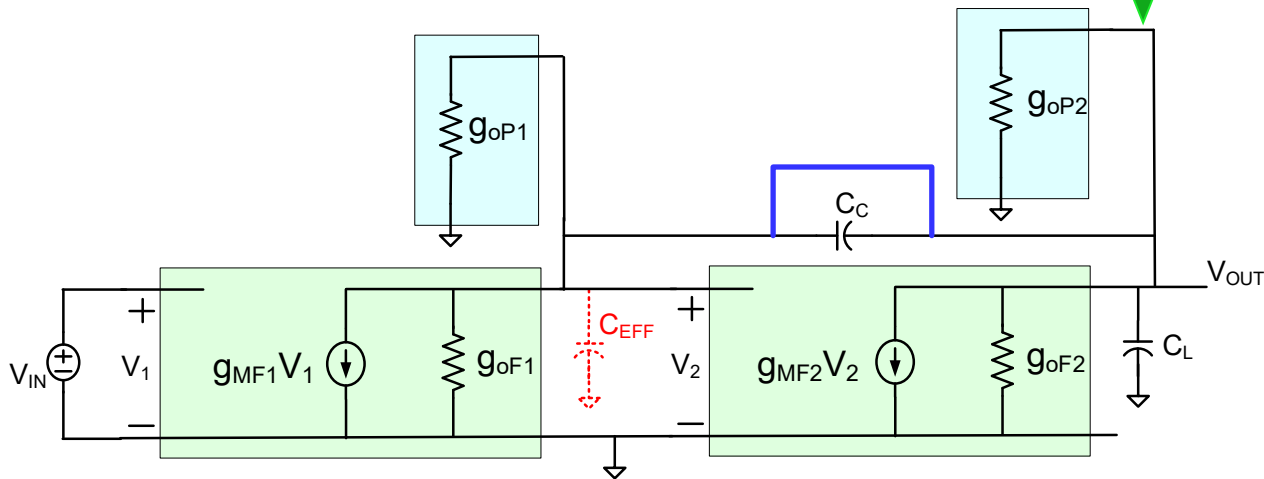
$$|p_1| = \frac{(g_{oF1} + g_{oP1})}{C_c} \longrightarrow |p_1| = (g_{oF1} + g_{oP1}) \left(\frac{g_{oF2} + g_{oP2}}{C_c g_{mF2}} \right)$$

$$\mathbf{BW} = |p_1|$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



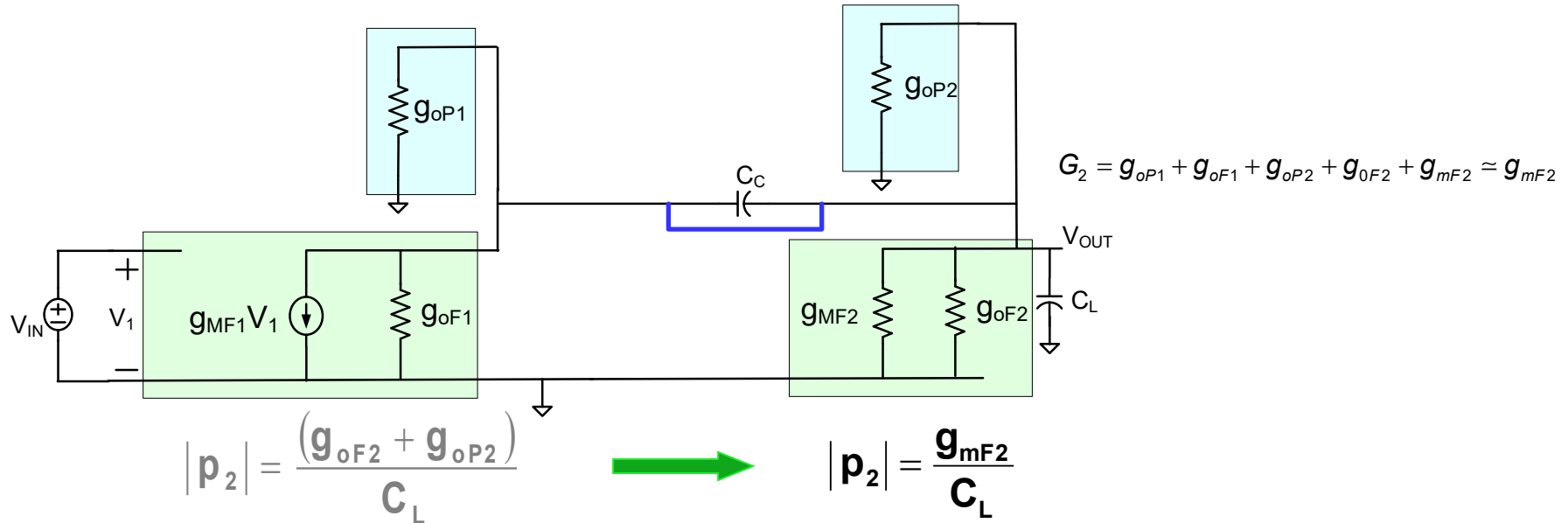
- To find the high-frequency pole p_2 , the circuit has changed
- At high frequencies C_C looks like a short circuit



Note the F2 block is now “diode connected” at high frequencies

$$G_2 = g_{oP1} + g_{oF1} + g_{oP2} + g_{oF2} + g_{mF2} \approx g_{mF2}$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



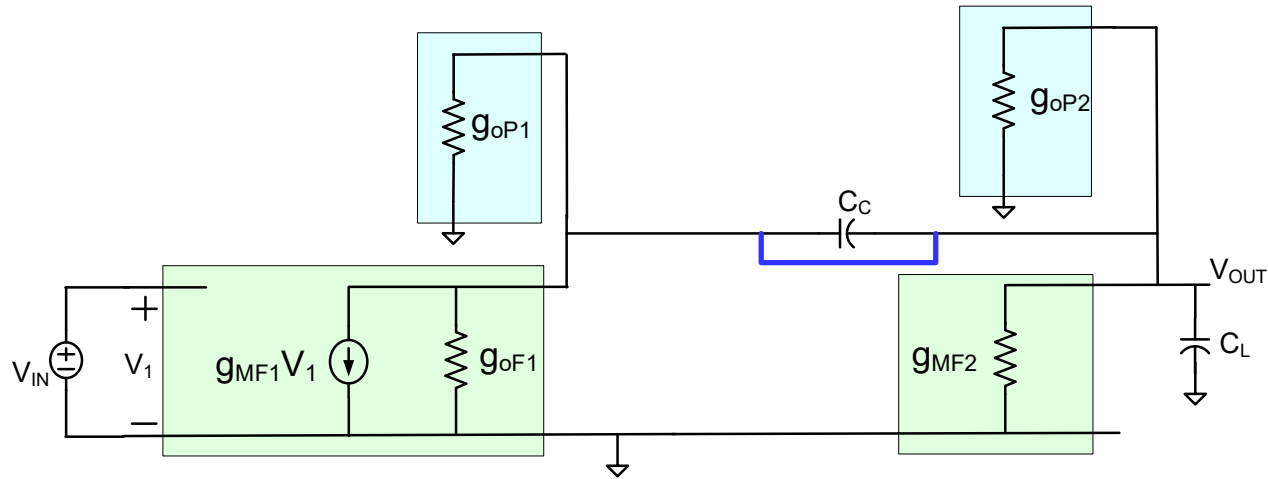
Will be shown later that C_C introduces a zero in the gain function

$$A_{v0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}} \right) \left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}} \right) \quad BW = (g_{oF1} + g_{oP1}) \left(\frac{g_{oF2} + g_{oP2}}{C_C g_{mF2}} \right) = \frac{g_{oF1} + g_{oP1}}{C_{EFF}}$$

$$GB = \frac{g_{mF1} g_{mF2}}{(g_{oF2} + g_{oP2}) C_C} \quad \xrightarrow{\text{If zero does not affect GB}} \quad GB = \frac{g_{mF1}}{C_C}$$

$$A(s) \simeq \frac{\left(\frac{s}{z_1} + 1 \right) g_{mF1} g_{mF2}}{s^2 C_C C_L + s C_C g_{mF2} + (g_{oF1} + g_{oP1})(g_{oF2} + g_{oP2})}$$

Pole Analysis of Internally Miller-Compensated Two-Stage Op Amps



$$|p_2| = \frac{(g_{oF2} + g_{oP2})}{C_L} \quad \longrightarrow \quad |p_2| = \frac{g_{mF2}}{C_L} \quad |p_2| \text{ has increased !}$$

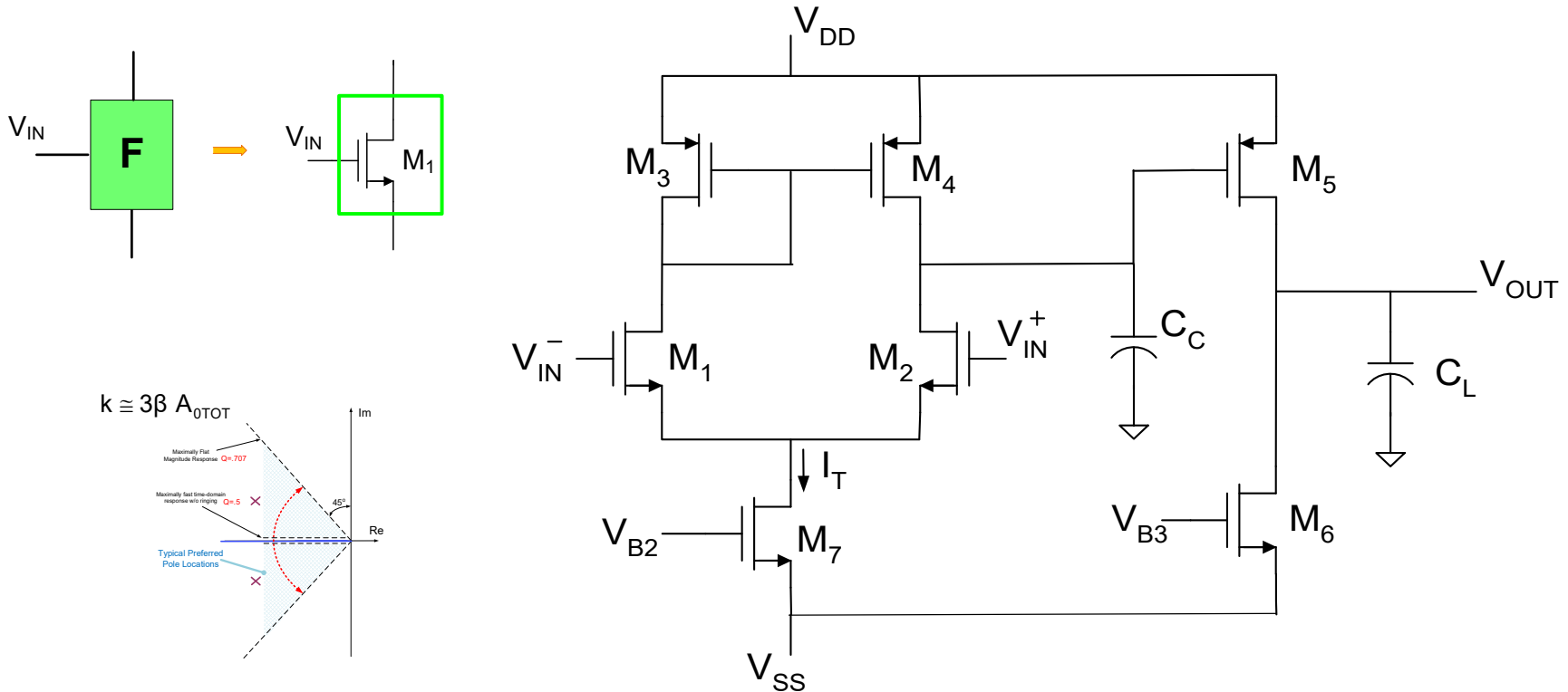
$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}} \right) \left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}} \right) \quad BW = (g_{oF1} + g_{oP1}) \left(\frac{g_{oF2} + g_{oP2}}{C_C g_{mF2}} \right) = \frac{g_{oF1} + g_{oP1}}{C_{EFF}}$$

$$GB = \frac{g_{mF1} g_{mF2}}{(g_{oF2} + g_{oP2}) C_C} \quad \xrightarrow{\text{If zero does not affect GB}} \quad GB = \frac{g_{mF1}}{C_C}$$

Has the GB decreased with the Miller compensation?

No, because the C_C decreased by the same factor!

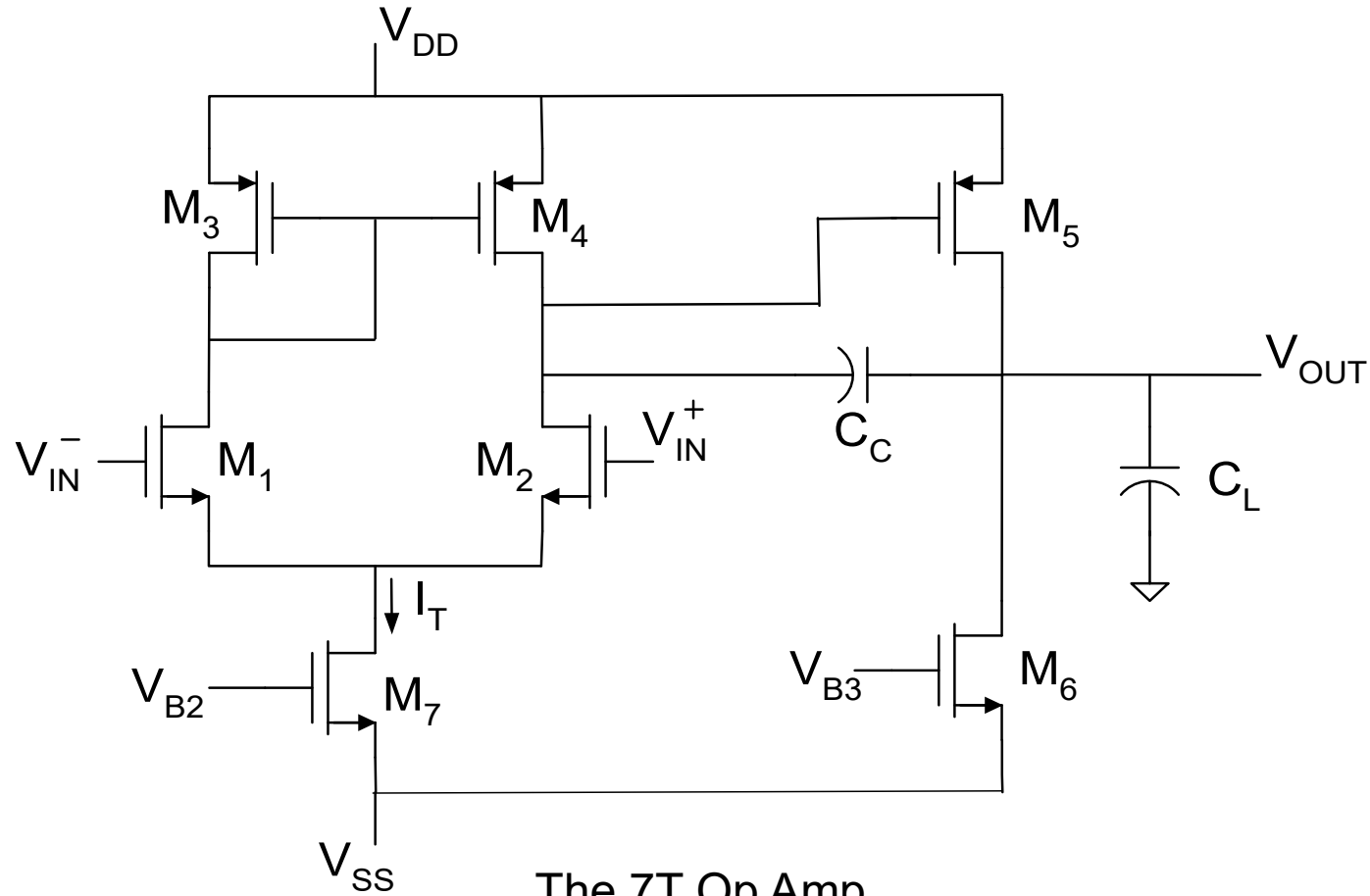
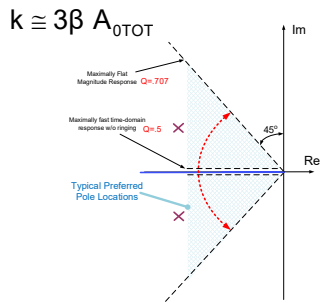
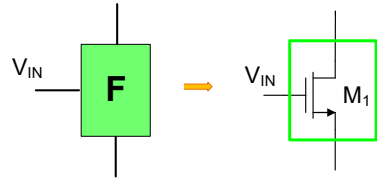
Basic Two-Stage Op Amp



- o Essentially just a cascade of two common-source stages
- o Same gain and pole expressions as developed for the cascade
- o Compensation Capacitor C_C used to get wide pole separation
- o Two poles in amplifier
- o No universally accepted strategy for designing this seemingly simple amplifier

Pole spread $\cong 3\beta A_{o1} A_{o2}$ makes C_C unacceptably large

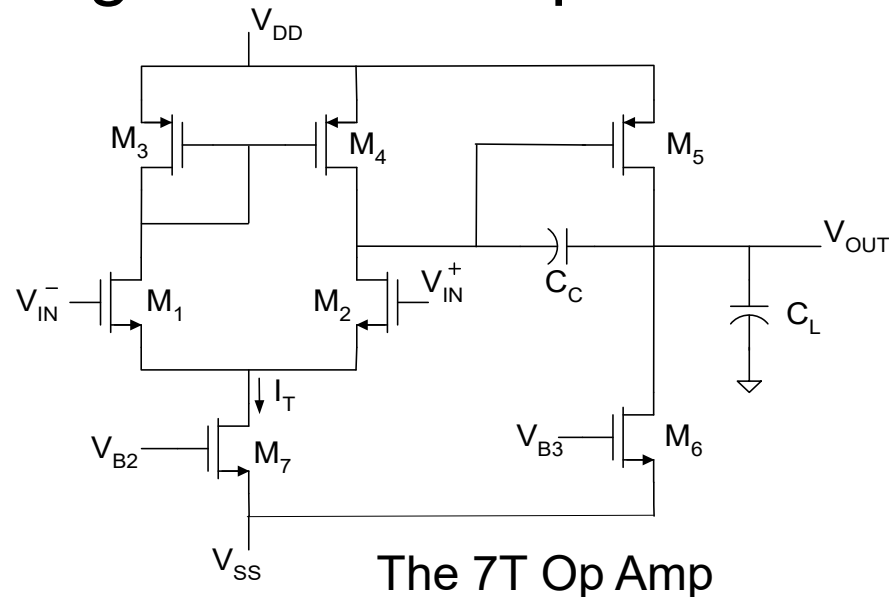
Basic Two-Stage Op Amp (with Miller Compensation)



The 7T Op Amp

- o Reduces C_C by approximately A_{02}
- o Pole spread $\approx 3\beta A_{01}A_{02}$ makes size of C_C manageable
- o One of the most widely used op amp architectures

Basic Two-Stage Miller Compensated Op Amp



By inspection (Notation: $p_1 = -\tilde{p}_1$ $p_2 = -\tilde{p}_2$)

$$\mathbf{A}_o = \left(\frac{-\mathbf{g}_{m1}}{\mathbf{g}_{o2} + \mathbf{g}_{o4}} \right) \left(\frac{\mathbf{g}_{m5}}{\mathbf{g}_{o5} + \mathbf{g}_{o6}} \right) \quad \tilde{p}_2 = \frac{\mathbf{g}_{m5}}{C_L}$$

$$\tilde{p}_1 = \frac{\mathbf{g}_{o2} + \mathbf{g}_{o4}}{C_C \left(\frac{\mathbf{g}_{m5}}{\mathbf{g}_{o5} + \mathbf{g}_{o6}} \right)}$$

If zero does not affect GB

$$\mathbf{GB} = \frac{\mathbf{g}_{m1}}{C_C}$$

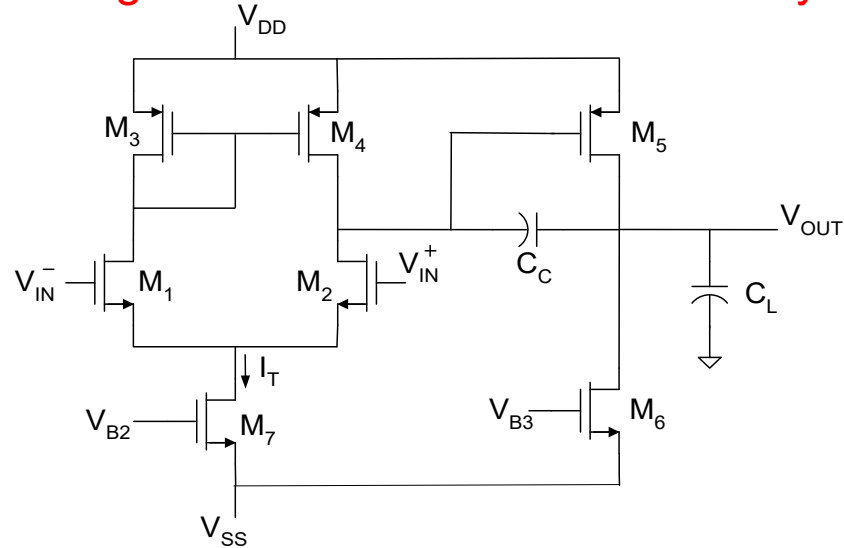
Will also get these results from a more complete (and time consuming) analysis

This analysis was based only upon finding the poles and will miss zeros if they exist

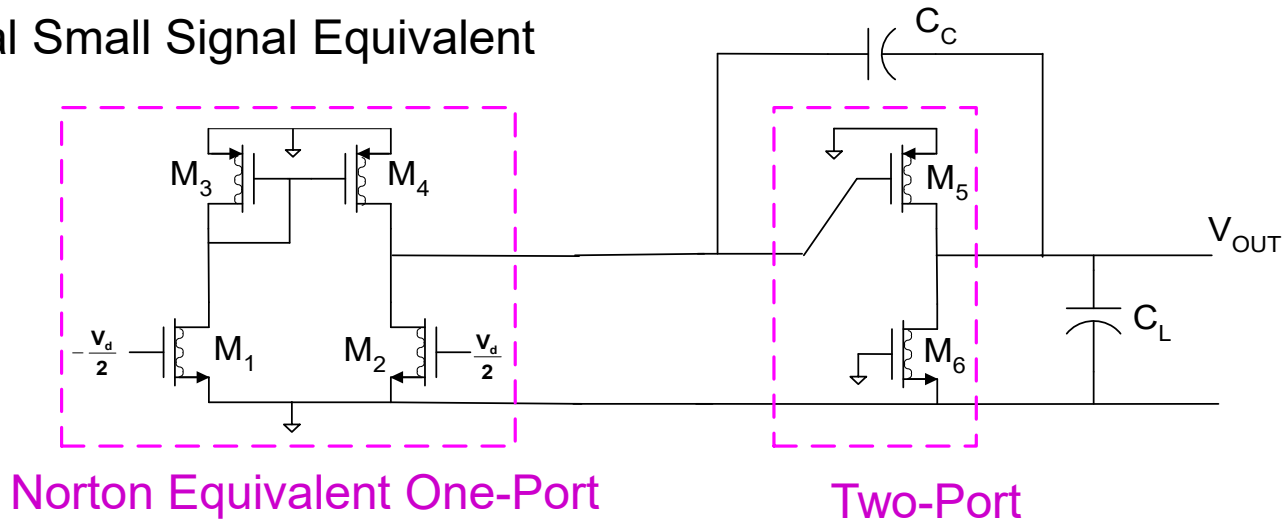
Small Signal Analysis of Basic Two-Stage Op Amp

(Will now obtain the actual gain which will show zeros if they exist)

(with Miller compensation)

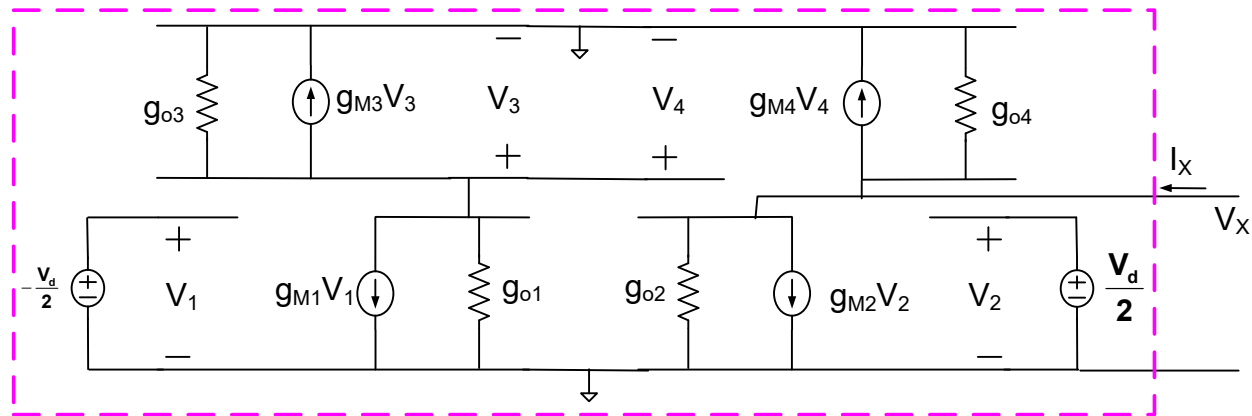
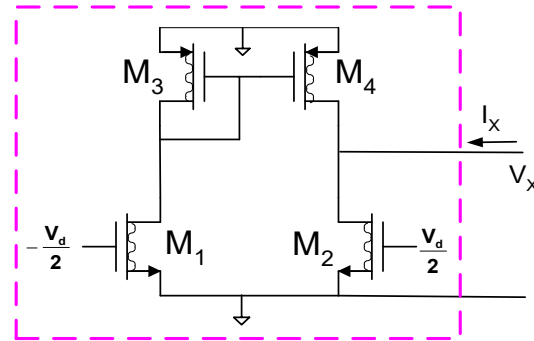


Differential Small Signal Equivalent



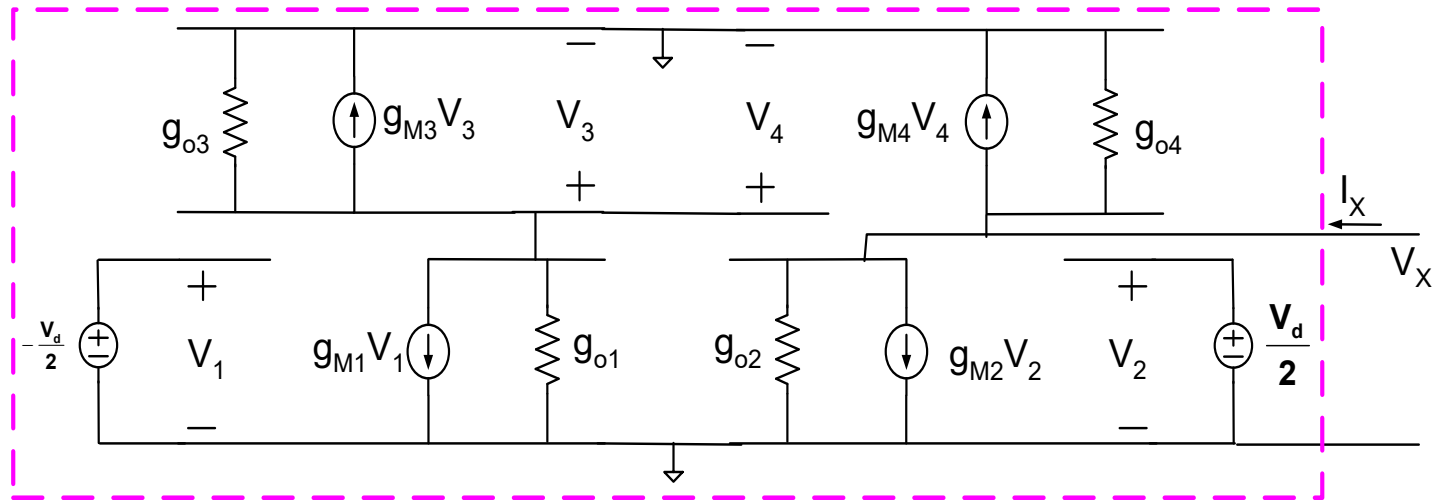
Small Signal Analysis of Basic Two-Stage Op Amp (with Miller compensation)

Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp (with Miller compensation)

Differential Small Signal Equivalent



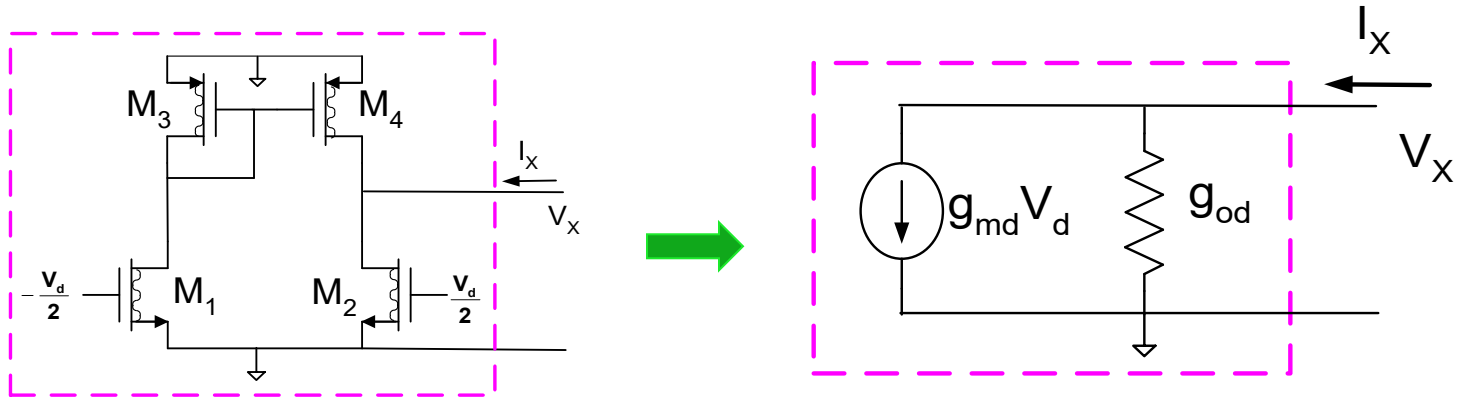
$$\left. \begin{aligned} I_X &= V_X (g_{o2} + g_{o4}) + g_{m2} \frac{V_d}{2} + g_{m4} V_4 \\ V_4 (g_{m3} + g_{o1} + g_{o3}) + g_{m1} \left(-\frac{V_d}{2} \right) &= 0 \end{aligned} \right\}$$

$$I_X = V_X (g_{o2} + g_{o4}) + g_{m2} V_d \left(\frac{1 + \frac{g_{m1}}{g_{m2}} \left(\frac{g_{m4}}{g_{m3} + g_{o2} + g_{o3}} \right)}{2} \right)$$

$$I_X \approx V_X (g_{o2} + g_{o4}) + g_{m2} V_d$$

Small Signal Analysis of Basic Two-Stage Op Amp (with Miller compensation)

Differential Small Signal Equivalent



$$I_x \cong V_x (g_{o2} + g_{o4}) + g_{m2} V_d$$

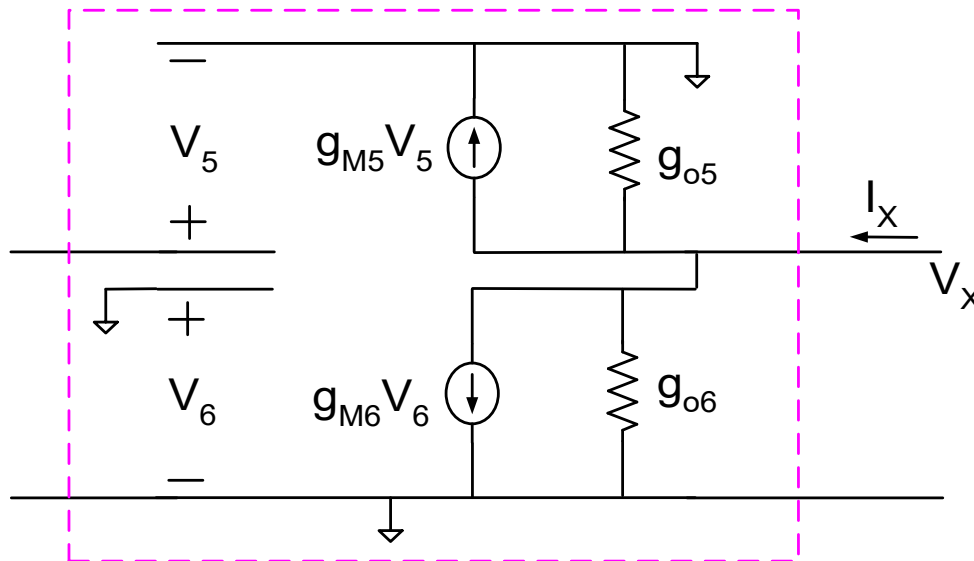
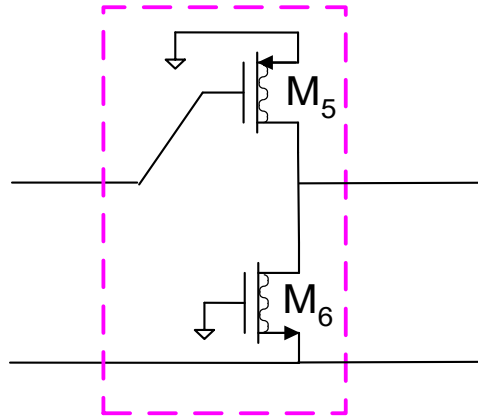
Since M_1 and M_2 are matched
as are M_3 and M_4

$$g_{md} = g_{m1}$$

$$g_{od} = g_{o2} + g_{o4}$$

Small Signal Analysis of Basic Two-Stage Op Amp (with Miller compensation)

Differential Small Signal Equivalent

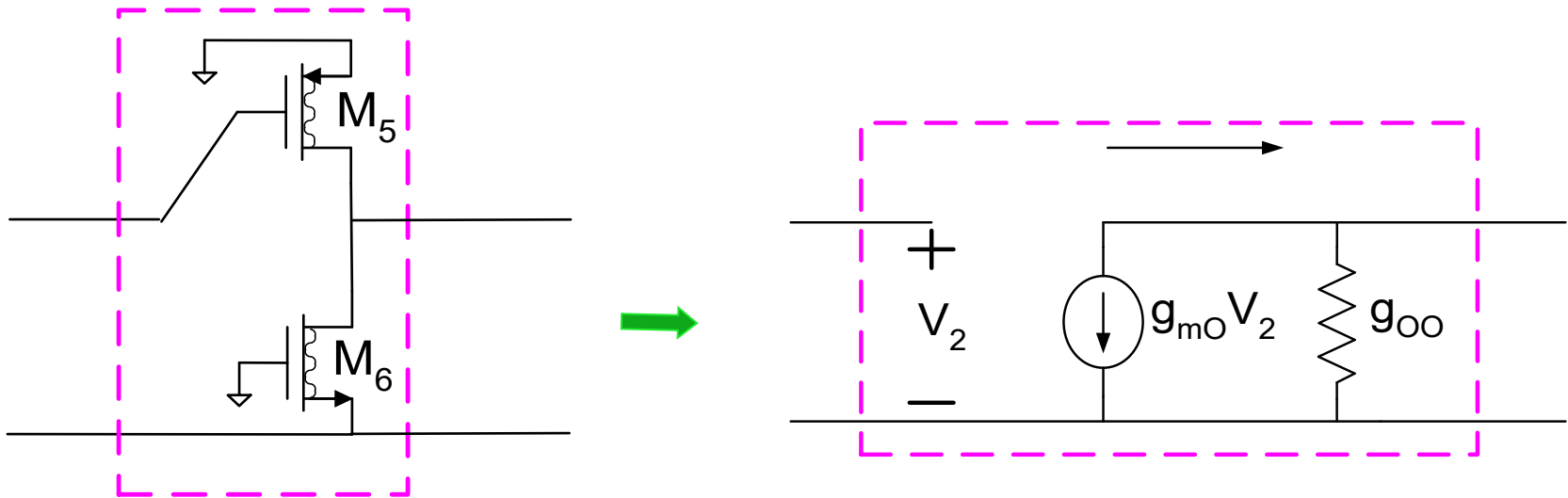


$$g_{oo} = g_{o5} + g_{o6}$$

$$g_{mo} = g_{m5}$$

Small Signal Analysis of Basic Two-Stage Op Amp (with Miller compensation)

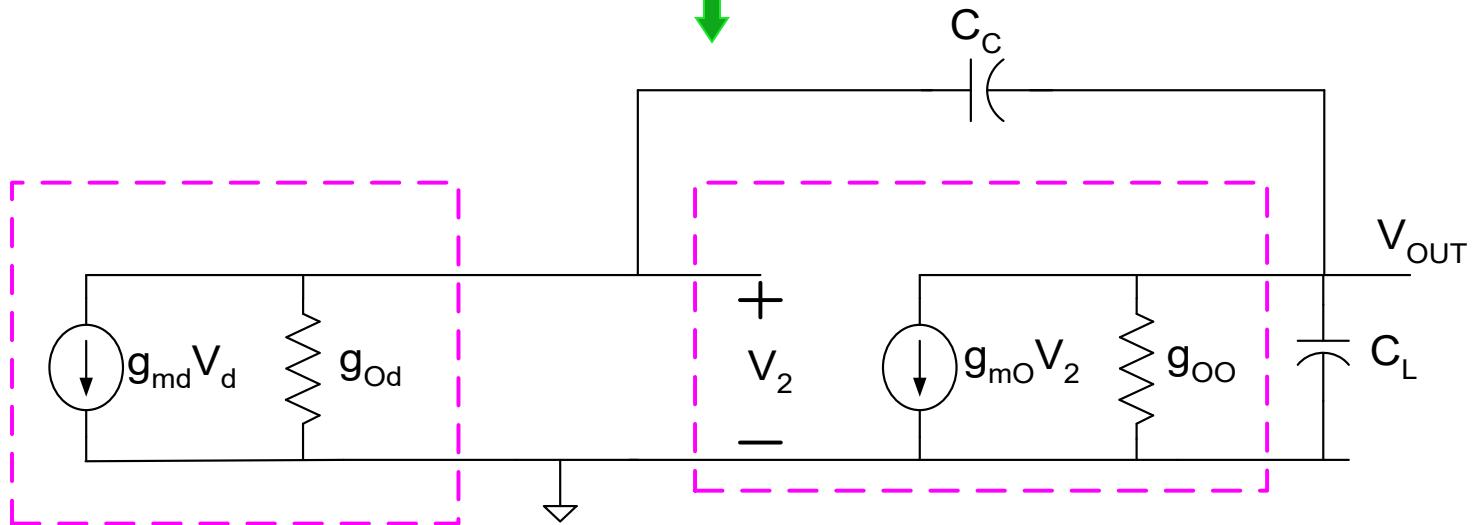
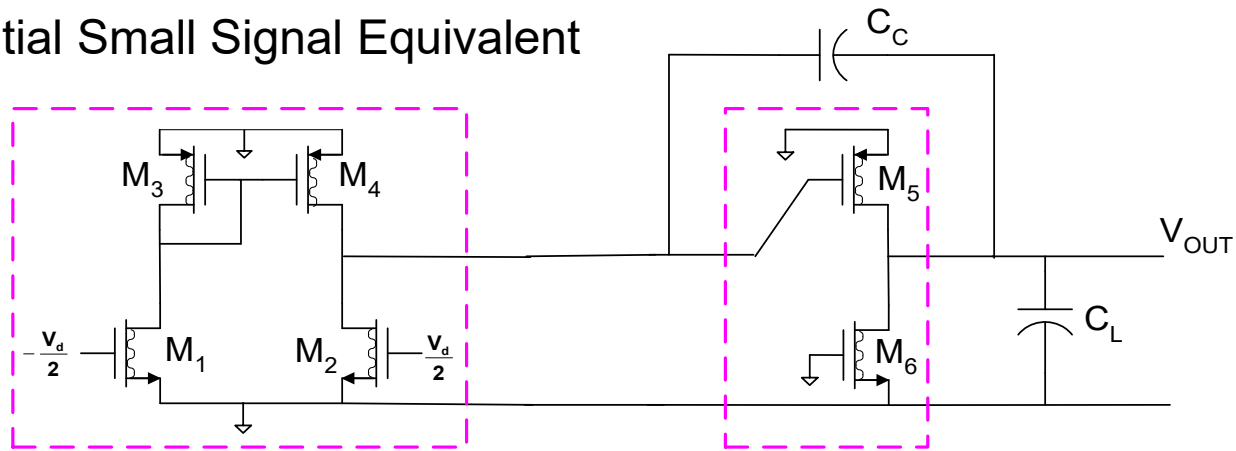
Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent

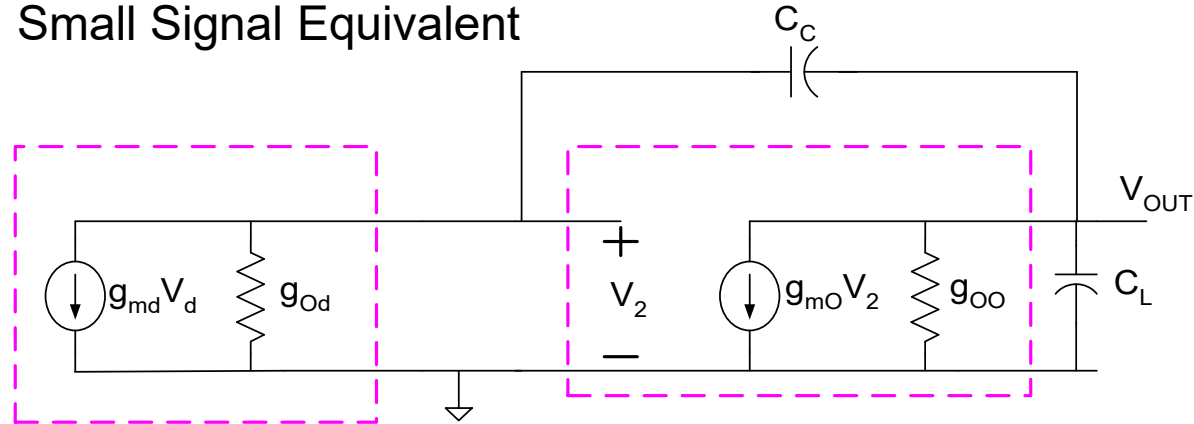


(This happens to be the general form for a two-stage structure with a quarter circuit and counterpart circuit !)

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent



$$\left. \begin{aligned} V_{\text{OUT}}(sC_C + sC_L + g_{oo}) + g_{mo} V_2 &= sC_C V_2 \\ V_2(sC_C + g_{od}) + g_{md} V_d &= sC_C V_{\text{OUT}} \end{aligned} \right\}$$

Solving we obtain:

$$V_{\text{OUT}} = V_d \frac{g_{md}(g_{mo} - sC_C)}{s^2 C_C C_L + s[g_{mo} C_C + (C_C(g_{oo} + g_{od}) + C_L g_{od})] + g_{oo} g_{od}}$$

This simplifies to:

$$V_{\text{OUT}} \cong V_d \frac{g_{md}(g_{mo} - sC_C)}{s^2 C_C C_L + s g_{mo} C_C + g_{oo} g_{od}}$$

(This happens to be the general form for a two-stage structure with a quarter circuit and counterpart circuit !)

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Differential Small Signal Equivalent

Summary:

$$A(s) = \frac{g_{md} (g_{mo} - sC_C)}{s^2 C_C C_L + s g_{mo} C_C + g_{oo} g_{od}}$$

where for the 7T implementation

$$g_{md} = g_{m1} = g_{m2}$$

$$g_{m0} = g_{m5}$$

$$g_{od} = g_{o2} + g_{o4}$$

$$g_{oo} = g_{o5} + g_{o6}$$

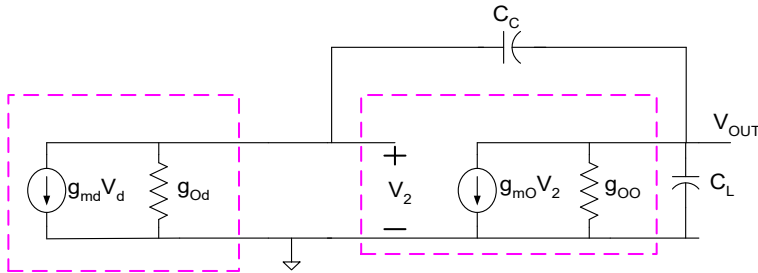
Note presence of single RHP zero!

How does this compare to the approximate analysis that obtained only the poles?

Small Signal Analysis of Basic Two-Stage Op Amp

(with Miller compensation)

Detailed analysis



$$A(s) = \frac{g_{md}(g_{mo} - sC_C)}{s^2 C_C C_L + s g_{mo} C_C + g_{oo} g_{od}}$$

$$z_1 = \frac{g_{mo}}{C_C}$$

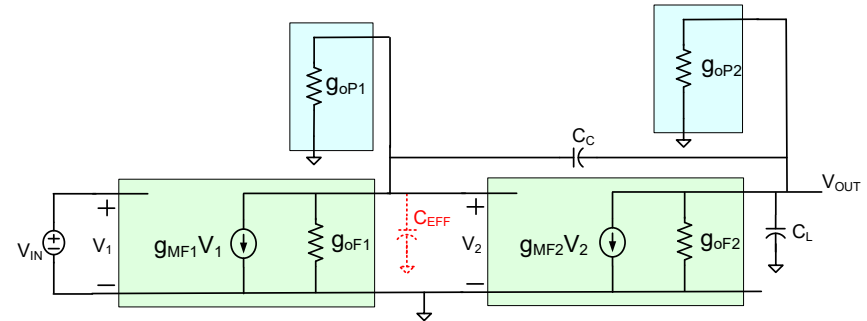
$$g_{md} = g_{mF1}$$

$$g_{m0} = g_{mF2}$$

$$g_{od} = g_{oF1} + g_{oP1}$$

$$g_{oo} = g_{oF2} + g_{oP2}$$

Inspection Analysis



$$A(s) \simeq \frac{\left(\frac{s}{z_1} + 1\right) g_{mF1} g_{mF2}}{s^2 C_C C_L + s C_C g_{mF2} + (g_{oF1} + g_{oP1})(g_{oF2} + g_{oP2})}$$

$$p_1 = -\frac{(g_{oF1} + g_{oP1})(g_{oF2} + g_{oP2})}{C_C g_{mF2}}$$

$$p_2 = -\frac{g_{mF2}}{C_L}$$

Same denominator so same poles and also same dc gain !

Small Signal Analysis of Two-Stage Miller-Compensated Op Amp

(with Miller compensation)

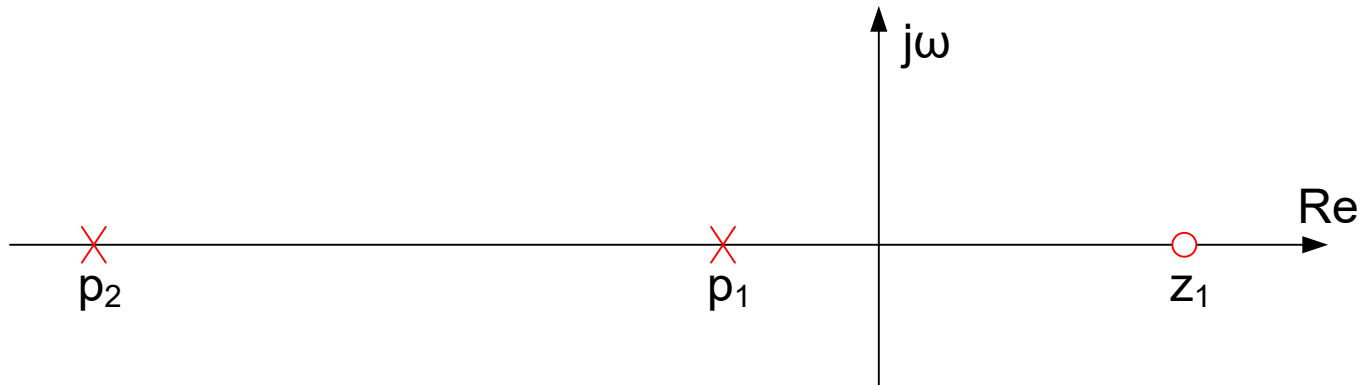
$$A(s) = \frac{g_{md} (g_{m0} - sC_C)}{s^2 C_C C_L + s g_{m0} C_C + g_{oo} g_{od}}$$

Note this is of the form:

(Notation: $p_1 = -\tilde{p}_1$ $p_2 = -\tilde{p}_2$ $z_1 = -\tilde{z}_1$)

$$A(s) = A_0 \frac{\frac{s}{\tilde{z}_1} + 1}{\left(\frac{s}{\tilde{p}_1} + 1\right) \left(\frac{s}{\tilde{p}_2} + 1\right)}$$

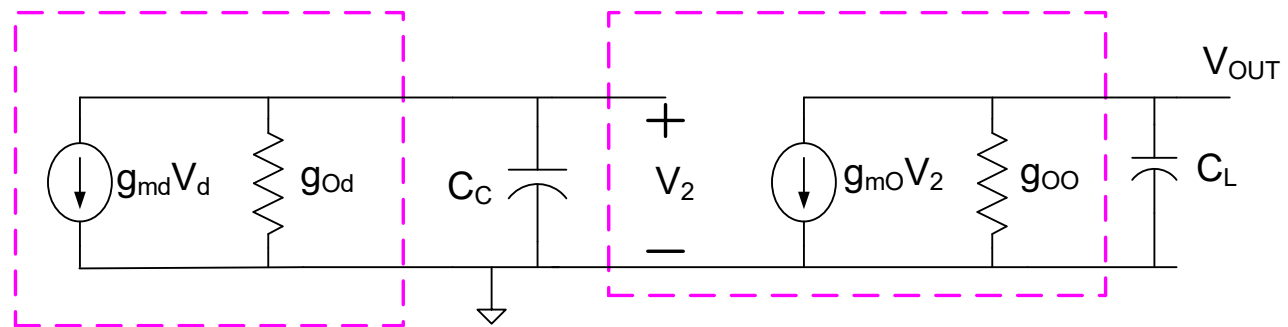
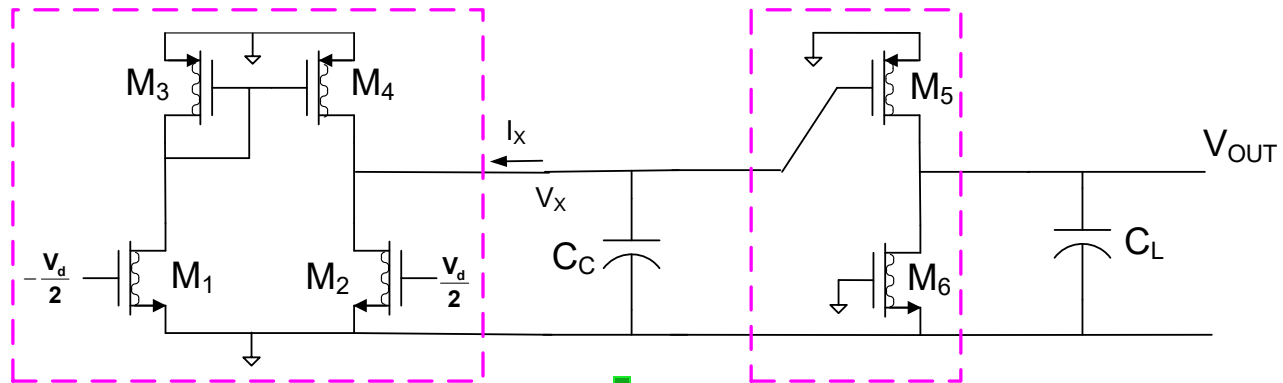
This has two negative real-axis poles and one positive real-axis zero



Small Signal Analysis of Basic Two-Stage Op Amp

(with Internal node compensation i.e. not Miller compensation)

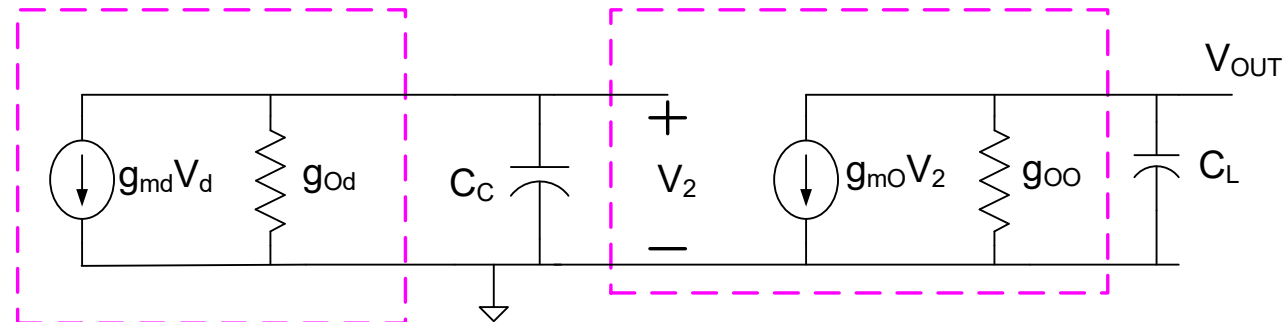
Differential Small Signal Equivalent



Small Signal Analysis of Basic Two-Stage Op Amp

(with Internal node compensation)

Differential Small Signal Equivalent



$$\left. \begin{aligned} V_{OUT} (sC_L + g_{00}) + g_{m0} V_2 &= 0 \\ V_2 (sC_C + g_{0d}) + g_{md} V_d &= 0 \end{aligned} \right\}$$

Solving we obtain:

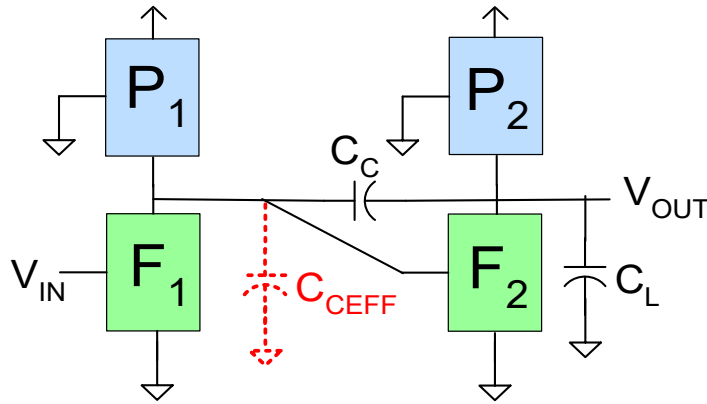
$$V_{OUT} = V_d \frac{g_{m0} g_{md}}{(sC_L + g_{00})(sC_C + g_{0d})}$$

This can be approximated by :

$$V_{OUT} = V_d \frac{g_{m0} g_{md}}{s^2 C_C C_L + s C_C g_{00} + g_{00} g_{0d}}$$

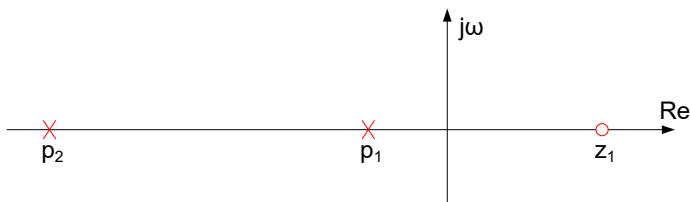
Can show this is the same as was obtained by inspection !

How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp?

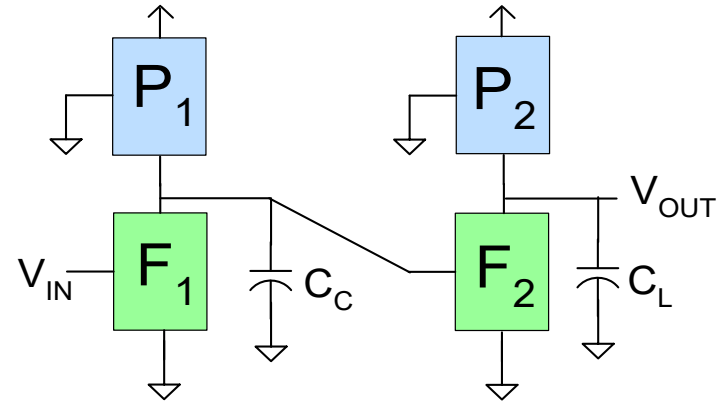


$$A(s) = \frac{g_{md}(g_{m0} - sC_C)}{s^2 C_C C_L + s g_{m0} C_C + g_{oo} g_{od}}$$

$$A(s) = A_0 \frac{\frac{s}{\tilde{z}_1} + 1}{\left(\frac{s}{\tilde{p}_1} + 1\right)\left(\frac{s}{\tilde{p}_2} + 1\right)}$$

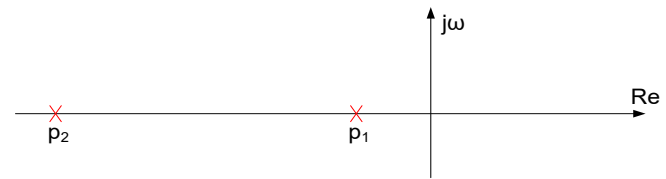


must be developed



$$A(s) \cong \frac{g_{md} g_{m0}}{s^2 C_C C_L + s C_C g_{oo} + g_{oo} g_{od}}$$

$$A(s) = A_0 \frac{1}{\left(\frac{s}{\tilde{p}_1} + 1\right)\left(\frac{s}{\tilde{p}_2} + 1\right)}$$



Compensation criteria:

$$4\beta A_0 > \frac{p_2}{p_1} > 2\beta A_0$$

Simple pole calculations for 2-stage op amp

Since the poles of the 2-stage op amp must be widely separated, a simple calculation of the poles from the characteristic polynomial is possible.

Assume p_1 and p_2 are the poles and $|p_1| \ll |p_2|$

$$D(s) = s^2 + a_1s + a_0$$

but

$$D(s) = (s - p_1)(s - p_2) = s^2 - s(p_1 + p_2) + p_1p_2 \approx \boxed{s^2 - p_2s} + p_1p_2$$

determines p_1

determines p_2

thus

$$p_2 = -a_1 \quad \text{and} \quad p_1 = -a_0/a_1$$

Example

A feedback amplifier has a characteristic polynomial of

$$D(s) = s^2 + 9000s + 1.8E3$$

Without using the quadratic equation, determine the poles by inspection and determine the ratio of the two poles.

solution

A feedback amplifier has a characteristic polynomial of

$$D(s) = s^2 + 9000s + 1.8E3$$

$$D(s) = \boxed{s^2 + 9000s} + 1.8E3$$

$$P_h = -9000$$

$$D(s) = s^2 + \boxed{9000s + 1.8E3}$$

$$P_L = -2$$

$$\text{Ratio} = 4500$$

Can now use these results to calculate poles of Basic Two-stage Miller Compensated Op Amp

From small signal analysis:

$$A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2 C_C C_L + s g_{m5} C_C + g_{oo} g_{od}}$$

$p_2 = -\frac{g_{m5}}{C_L}$
 $p_1 = -\frac{g_{oo} g_{od}}{g_{m5} C_C}$

$$g_{md} = g_{m1} = g_{m2}$$

$$g_{od} = g_{o2} + g_{o4}$$

$$g_{oo} = g_{o5} + g_{o6}$$

$$A_0 = \frac{g_{m5} g_{md}}{g_{oo} g_{od}}$$

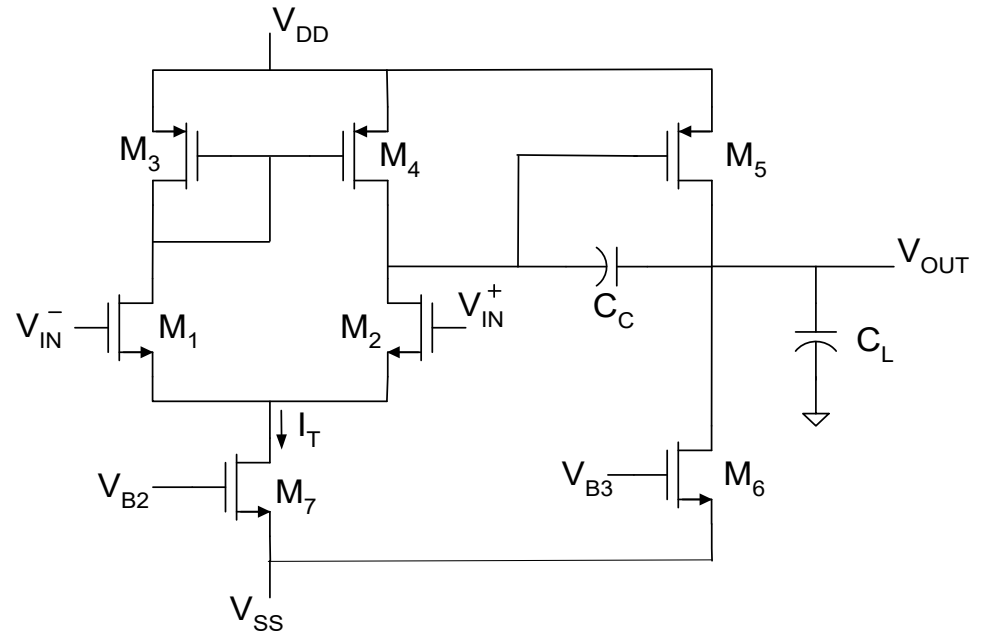
$$GB = \frac{g_{m5} g_{md}}{g_{oo} g_{od}} \cdot |p_1| = \frac{g_{m5} g_{md}}{g_{oo} g_{od}} \cdot \frac{g_{oo} g_{od}}{g_{m5} C_C} = \frac{g_{md}}{C_C}$$

From Previous Inspection

$$\mathbf{A}_o = \left(\frac{-\mathbf{g}_{m1}}{\mathbf{g}_{o2} + \mathbf{g}_{o4}} \right) \left(\frac{\mathbf{g}_{m5}}{\mathbf{g}_{o5} + \mathbf{g}_{o6}} \right)$$

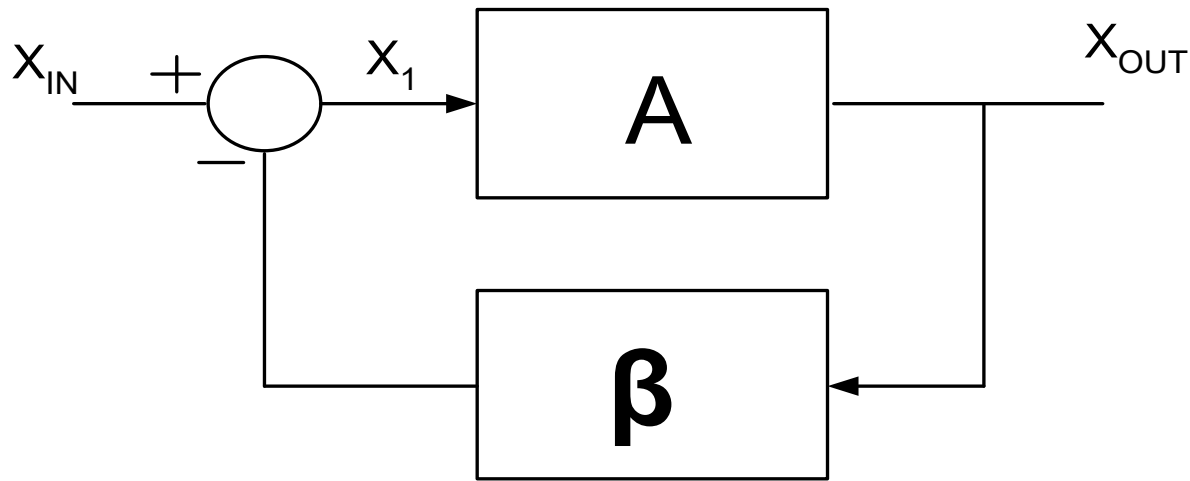
$$p_1 = -\frac{\mathbf{g}_{o2} + \mathbf{g}_{o4}}{C_C \left(\frac{\mathbf{g}_{m5}}{\mathbf{g}_{o5} + \mathbf{g}_{o6}} \right)} \quad p_2 = -\frac{\mathbf{g}_{m5}}{C_L}$$

$$\mathbf{GB} = \frac{\mathbf{g}_{m1}}{C_C}$$



Note the simple results obtained from inspection agree with the more time consuming results obtained from a small signal analysis

Feedback applications of the two-stage Op Amp



How does the amplifier perform with feedback ?

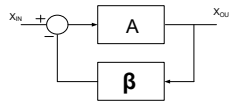
How should the amplifier be compensated?

Feedback applications of the two-stage Op Amp

Open-loop Gain

$$A(s) = \frac{N(s)}{D(s)}$$

Standard Feedback Gain



$$A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{N(s)}{D(s) + N(s)\beta(s)} \stackrel{\text{def n}}{=} \frac{N_{FB}(s)}{D_{FB}(s)}$$

$$N_{FB}(s) = N(s)$$

$$D_{FB}(s) = D(s) + \beta(s)N(s)$$

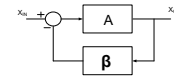
- Open-loop and closed-loop zeros identical (for standard feedback gain)
- Closed-loop poles different than open-loop poles
- Often $\beta(s)$ is not dependent upon frequency
- Open-loop zeros, gain, and β play a key role in determining closed-loop poles

Feedback applications of the two-stage Op Amp

Open-loop Gain

$$A(s) = \frac{N(s)}{D(s)}$$

Standard Feedback Gain



$$A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{\frac{1}{\beta(s)}}{1 + \frac{1}{A(s)\beta(s)}}$$

Alternate Feedback Gain (often FB is not of “standard” form)

$$A_{FB}(s) = \frac{\frac{1}{\beta_1(s)}}{1 + \frac{1}{A(s)\beta(s)}} = \frac{\frac{\beta(s)}{\beta_1(s)} N(s)}{D(s) + N(s)\beta(s)}$$

In either case, denominators are the same and characteristic equation defined by

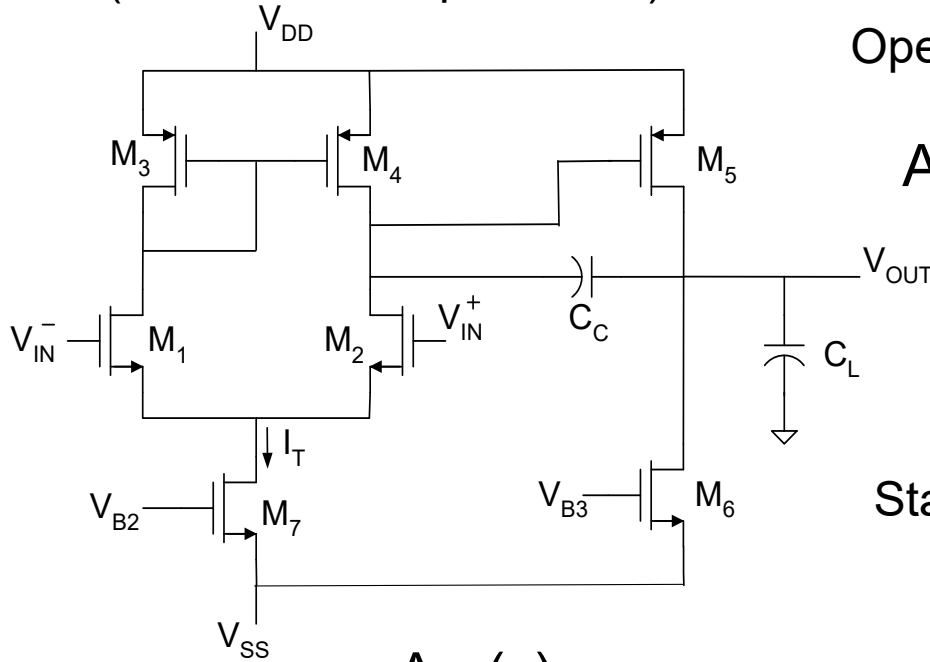
$$D_{FB}(s) = D(s) + \beta(s)N(s)$$

Often $\beta(s)$ and $\beta_1(s)$ are not dependent upon frequency and in this case

$$N_{FB}(s) = N(s)$$

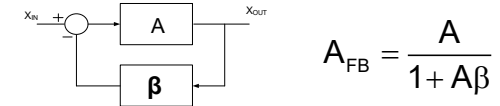
Basic Two-Stage Op Amp with Feedback

(with Miller compensation)



Open-loop gain

$$A(s) = \frac{g_{md}(g_{mo} - sC_c)}{s^2 C_c C_L + s C_c g_{mo} + g_{oo} g_{od}}$$



Standard feedback gain with constant β

$$A_{FB}(s) = \frac{g_{md}(g_{mo} - sC_c)}{s^2 C_c C_L + s C_c (g_{mo} - \beta g_{md}) + g_{oo} g_{od} + \beta g_{md} g_{mo}}$$

$$A_{FB}(s) \cong \frac{g_{md}(g_{mo} - sC_c)}{s^2 C_c C_L + s C_c (g_{mo} - \beta g_{md}) + \beta g_{md} g_{mo}}$$

where

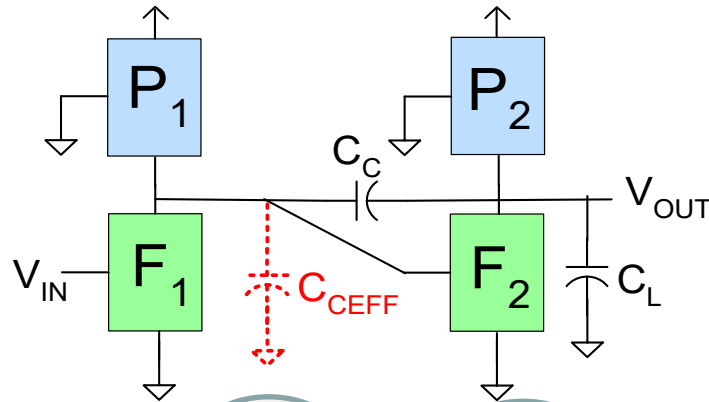
$$g_{md} = g_{m1}$$

$$g_{mo} = g_{m5}$$

$$g_{od} = g_{o2} + g_{o4}$$

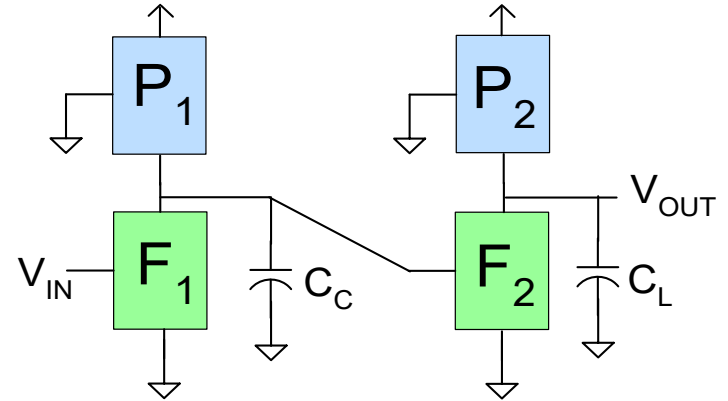
$$g_{oo} = g_{o5} + g_{o6}$$

How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp with feedback $A_{FB} = \frac{A}{1+A\beta}$?



$$A(s) = \frac{g_{md}(g_{m0} - sC_C)}{s^2 C_C C_L + s g_{m0} C_C + g_{oo} g_{od}}$$

$$A_{FB}(s) \cong \frac{g_{md}(g_{m0} - sC_c)}{s^2 C_C C_L + s C_C (g_{m0} - \beta g_{md}) + \beta g_{md} g_{m0}}$$

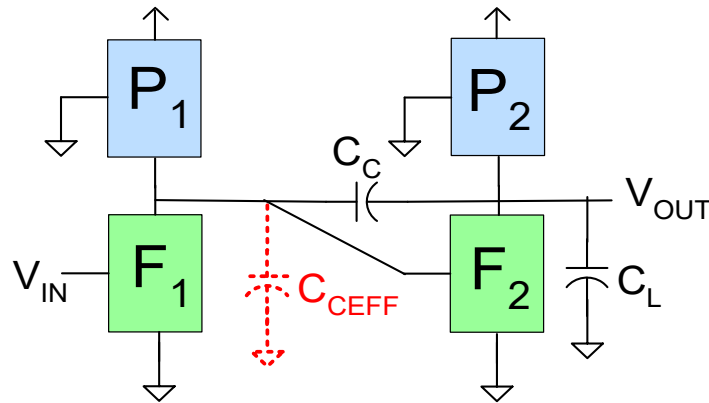


$$A(s) \cong \frac{g_{md} g_{m0}}{s^2 C_C C_L + s C_C g_{oo} + g_{oo} g_{od}}$$

$$A_{FB}(s) \cong \frac{g_{m0} g_{md}}{s^2 C_C C_L + s C_C g_{oo} + \beta g_{m0} g_{md}}$$

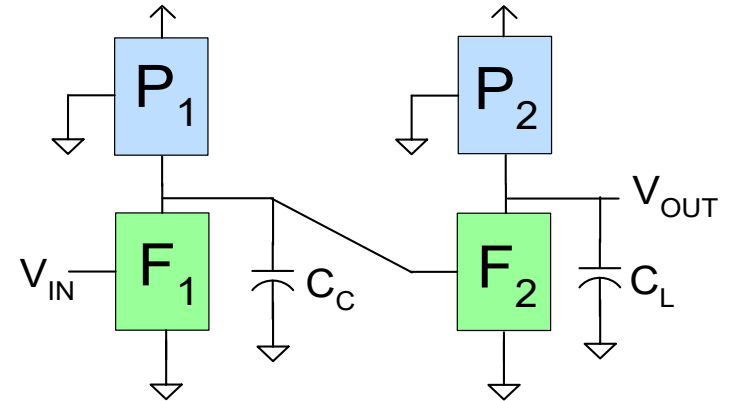
Zero in open-loop gain introduces the $-\beta g_{md}$ term in FB configuration

How was compensation done before the work of Fullagar ?



$$A(s) = \frac{g_{md}(g_{m0} - sC_C)}{s^2 C_C C_L + s g_{m0} C_C + g_{oo} g_{od}}$$

$$A_{FB}(s) \cong \frac{g_{md}(g_{m0} - sC_c)}{s^2 C_c C_L + s C_c (g_{m0} - \beta g_{md}) + \beta g_{md} g_{m0}}$$



$$A(s) \cong \frac{g_{md} g_{m0}}{s^2 C_C C_L + s C_C g_{oo} + g_{oo} g_{od}}$$

$$A_{FB}(s) \cong \frac{g_{m0} g_{md}}{s^2 C_C C_L + s C_C g_{oo} + \beta g_{m0} g_{md}}$$

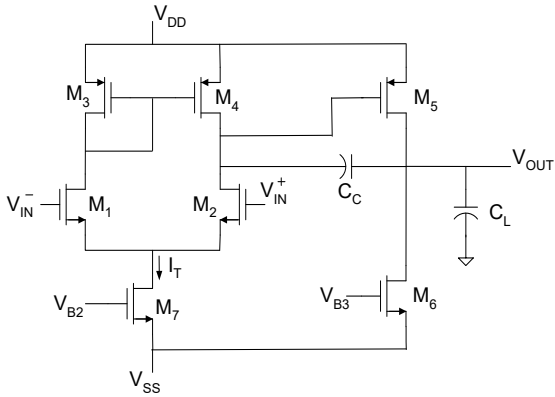
Internal node capacitor C_C or Miller C_C added externally

Or “load compensation” before output buffer added externally

Termed “externally compensated”

Basic Two-Stage Op Amp

(with Miller compensation) $A_{FB} = \frac{A}{1+A\beta}$



$$A_{FB}(s) \cong \frac{g_{md}(g_{mo} - sC_c)}{s^2 C_c C_L + s C_c (g_{mo} - \beta g_{md}) + \beta g_{md} g_{mo}}$$

Pole Q = ?

Review of Basic Concepts

Consider a second-order factor of a denominator polynomial, $P(s)$, expressed in integer-monic form

$$P(s) = s^2 + a_1s + a_0$$

Then $P(s)$ can be expressed in several alternative but equivalent ways

$$s^2 + s \frac{\omega_0}{Q} + \omega_0^2$$

$$s^2 + s2\zeta\omega_0 + \omega_0^2$$

$$(s - p_1)(s - p_2)$$

and if complex conjugate poles,

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$

$$(s - re^{j\theta})(s - re^{-j\theta})$$

and if negative real – axis poles

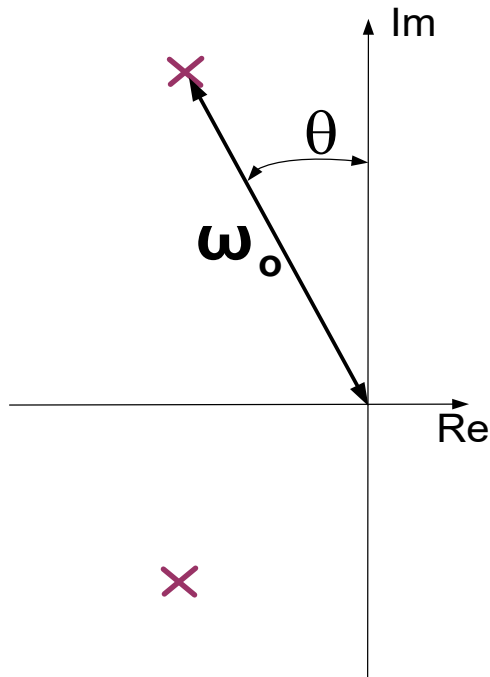
$$(s - p_1)(s - kp_1)$$

These are 7 different 2-parameter characterizations of the second-order factor and it is easy to map from any one characterization to any other !

$$\{a_1 \ a_0\} \ \{\omega_0 \ Q\} \ \{\omega_0 \ \zeta\} \ \{p_1 \ p_2\} \ \{\alpha \ \beta\} \ \{r \ \theta\} \ \{p_1 \ k\}$$

Review of Basic Concepts

$$s^2 + s \frac{\omega_0}{Q} + \omega_0^2$$



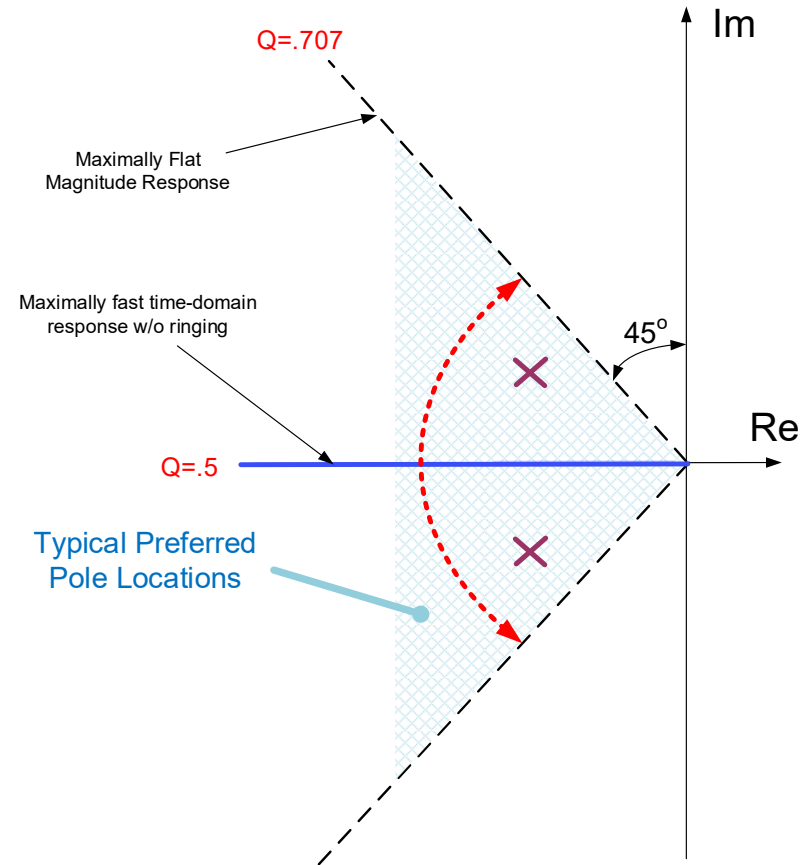
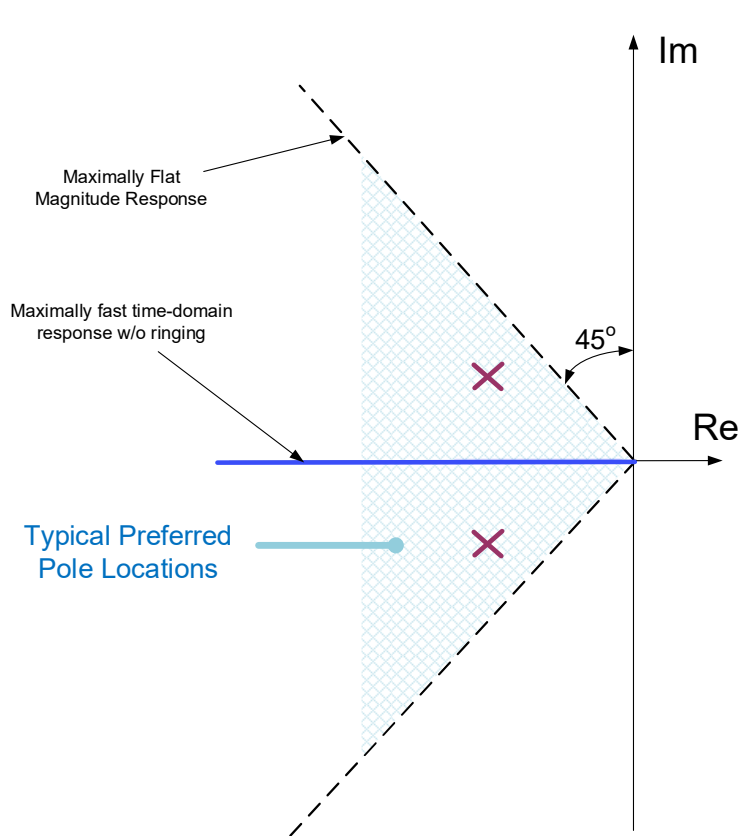
$$\sin\theta = \frac{1}{2Q}$$

ω_0 = magnitude of pole

Q determines the angle of the pole

Observe: $Q=0.5$ corresponds to two identical real-axis poles
 $Q=.707$ corresponds to poles making 45° angle with Im axis

What closed-loop pole Q is typically required when compensating an op amp?



Recall:

Typically compensate so closed-loop poles make angle between 45° and 90° from imaginary axis

Equivalently:

$$0.5 < Q < .707$$

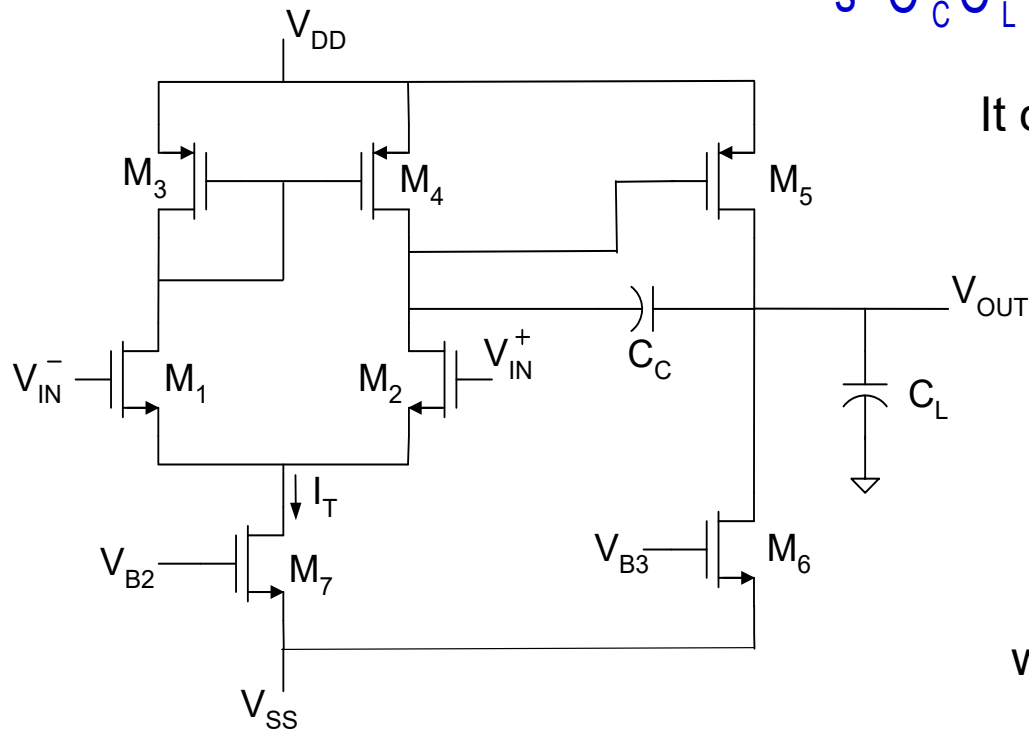
Basic Two-Stage Op Amp

(with Miller compensation)

Standard Feedback Gain



$$A_{FB}(s) \cong \frac{g_{md}(g_{mo} - sC_c)}{s^2 C_c C_L + sC_c(g_{mo} - \beta g_{md}) + \beta g_{md} g_{mo}}$$



It can be shown that

$$Q = \sqrt{\frac{C_L}{C_C}} \sqrt{\beta} \frac{\sqrt{g_{mo} g_{md}}}{g_{mo} - \beta g_{md}}$$

$$C_C = \frac{C_L \beta}{Q^2} \frac{g_{mo} g_{md}}{(g_{mo} - \beta g_{md})^2}$$

where $g_{md} = g_{m1}$ $g_{mo} = g_{m5}$

$g_{oo} = g_{o5} + g_{o6}$ and $g_{od} = g_{o2} + g_{o4}$

But what pole Q is desired? $.707 < Q < 0.5$

Right Half-Plane Zero in OL Gain (from Miller Compensation) Limits Performance

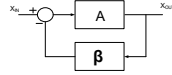
(because it increases the pole Q and thus requires a larger C_C!)

Closed-form expression for C_C!

Basic Two-Stage Op Amp

(with Miller compensation)

Standard Feedback Gain



$$A_{FB}(s) \cong \frac{g_{md}(g_{m0} - sC_c)}{s^2 C_c C_L + sC_c(g_{m0} - \beta g_{md}) + \beta g_{md} g_{m0}}$$

$$Q = \sqrt{\frac{C_L}{C_c}} \sqrt{\beta} \frac{\sqrt{g_{m0} g_{md}}}{g_{m0} - \beta g_{md}}$$

$$C_c = \frac{C_L \beta}{Q^2} \frac{g_{m0} g_{md}}{(g_{m0} - \beta g_{md})^2}$$

Question: Can we express C_c in terms of the pole spread k instead of in terms of Q ?

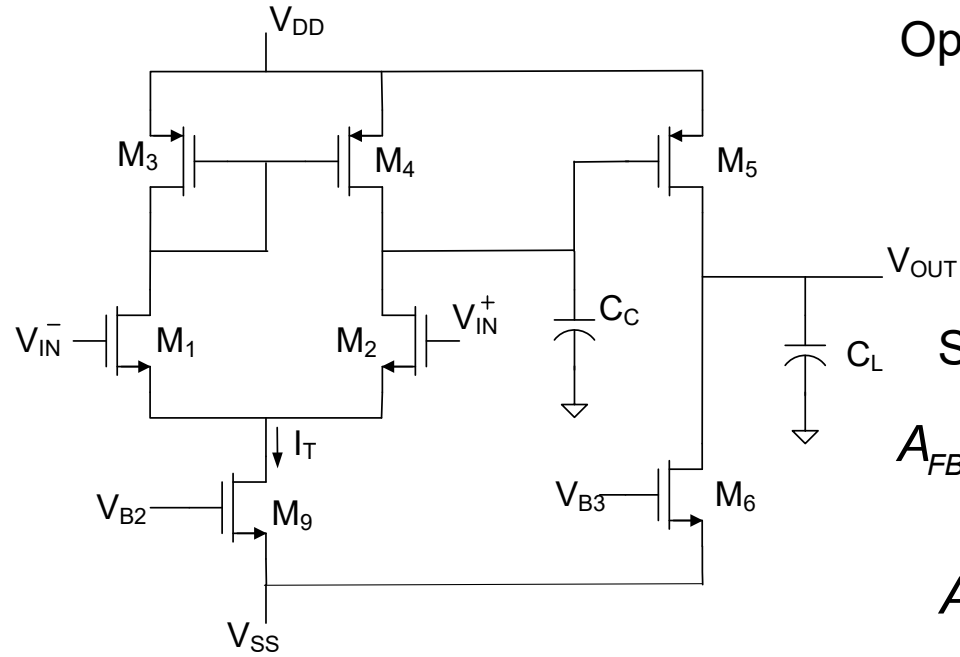
Recall when criteria $2\beta A_0 < k < 4\beta A_0$ was derived (Lect 13), started with expression:

$$Q = \frac{\sqrt{k}}{(1+k)} \sqrt{\beta A_{0TOT}} \underset{k \text{ large}}{\cong} \sqrt{\frac{\beta A_{0TOT}}{k}} \quad \longrightarrow \quad k \underset{k \text{ large}}{\cong} \frac{\beta A_{0TOT}}{Q^2}$$

No ! Relationship between k and Q was developed for 2nd-order lowpass open-loop gain (i.e. no zeros present!)

Basic Two-Stage Op Amp with Feedback

(with Internal Node compensation)



Open-loop gain

$$A(s) = \frac{g_{m0}g_{md}}{s^2C_C C_L + sC_C g_{o0} + g_{o0}g_{od}}$$

$$A_{FB} = \frac{A}{1 + A\beta}$$

Standard feedback gain with constant β

$$A_{FB}(s) = \frac{g_{m0}g_{md}}{s^2C_C C_L + sC_C g_{o0} + g_{o0}g_{od} + \beta g_{m0}g_{md}}$$

$$A_{FB}(s) \cong \frac{g_{m0}g_{md}}{s^2C_C C_L + sC_C g_{o0} + \beta g_{m0}g_{md}}$$

where

$$g_{oo} = g_{o5} + g_{o6}$$

$$g_{mo} = g_{m5}$$

$$g_{od} = g_{o2} + g_{o4}$$

$$g_{md} = g_{m1}$$

$$4\beta A_0 > \frac{p_2}{p_1} > 2\beta A_0 \iff k \cong \frac{\beta A_{0TOT}}{Q^2}$$

$$p_2 = \frac{g_{o0}}{C_L} \quad p_1 = \frac{g_{od}}{C_C} \quad A_0 = \frac{g_{m0}g_{md}}{g_{o0}g_{od}}$$

$$C_L 4\beta \frac{g_{m0}g_{md}}{g_{o0}^2} > C_C > C_L 2\beta \frac{g_{m0}g_{md}}{g_{o0}^2}$$

$$\text{OR } C_C = C_L \beta \frac{g_{m0}g_{md}}{Q^2 g_{o0}^2} = C_L \beta \frac{g_{m5}g_{m1}}{Q^2 (g_{o5} + g_{o6})^2}$$

Status on Compensation

Generally not needed for single-stage op amps

Analytical expressions were developed with $A_{FB} = \frac{A}{1+A\beta}$ for

Two-stage with internal node compensation (no OL zeros)

Two-stage with load compensation (no OL zeros)

Two-stage with basic Miller compensation (OL zero, single series comp cap)

Will now develop a more general compensation strategy



Stay Safe and Stay Healthy !

End of Lecture 15